

## Differentiation - Chain Rule

Differentiate each function with respect to  $x$ .

1)  $y = (x^3 + 3)^5$

$$y' = 5(x^3 + 3)^4 \cdot (3x^2)$$

2)  $y = (-3x^5 + 1)^3$

$$y' = 3(-3x^5 + 1)^2 \cdot (-15x^4)$$

3)  $y = (-5x^3 - 3)^3$

$$y' = 3(-5x^3 - 3)^2 \cdot (-15x^2)$$

4)  $y = (5x^2 + 3)^4$

$$y' = 4(5x^2 + 3)^3 \cdot (10x)$$

5)  $f(x) = \sqrt[4]{-3x^4 - 2}$

$$f(x) = (-3x^4 - 2)^{\frac{1}{4}}$$

$$f'(x) = \frac{1}{4}(-3x^4 - 2)^{-\frac{3}{4}} \cdot (-12x^3)$$

6)  $f(x) = \sqrt{-2x^2 + 1}$

$$f(x) = (-2x^2 + 1)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(-2x^2 + 1)^{-\frac{1}{2}} \cdot (-4x)$$

7)  $f(x) = \sqrt[3]{-2x^4 + 5}$

$$f(x) = (-2x^4 + 5)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}(-2x^4 + 5)^{-\frac{2}{3}} \cdot (-8x)$$

8)  $y = (-x^4 - 3)^{-2}$

$$y' = -2(-x^4 - 3)^{-3} \cdot (-4x^3)$$

$$9) y = (3x^3 + 1)(-4x^2 - 3)^4$$

$$y' = (3x^3 + 1)'(-4x^2 - 3)^4 + (3x^3 + 1)[(-4x^2 - 3)^4]'$$

$$= 9x^2(-4x^2 - 3)^4 + (3x^3 + 1) \cdot 4(-4x^2 - 3)^3 \cdot (-8x)$$

$$10) y = \frac{(x^3 + 4)^5}{3x^4 - 2}$$

$$y' = \frac{5(x^3 + 4)(3x^2)(3x^4 - 2) - (x^3 + 4)^5(12x^3)}{(3x^4 - 2)^2}$$

$$11) y = ((x+5)^5 - 1)^4 \quad \leftarrow 2 \text{ chain Rules}$$

$$y' = 4((x+5)^5 - 1)^3 \cdot 5(x+5)^4 \cdot 1$$

$$12) y = (5x^3 - 3)^5 \sqrt[4]{-4x^5 - 3}$$

$$y = (5x^3 - 3)^5 (-4x^5 - 3)^{\frac{1}{4}}$$

$$y' = 5(5x^3 - 3) \cdot (15x^2) (-4x^5 - 3)^{\frac{1}{4}} + (5x^3 - 3)^5 \cdot \frac{1}{4} (-4x^5 - 3)^{-\frac{3}{4}} \cdot (-20x^4)$$

**Critical thinking question:**

13) Give a function that requires three applications of the chain rule to differentiate. Then differentiate the function.

Similar to 11) above. 11) requires 2 chain rules

consider  $y = ((x+5)^5 - 1)^4 - 1)^2$

then  $y' = 2[(x+5)^5 - 1 - 1] \cdot 4((x+5)^5 - 1)^3 \cdot 5(x+5)^4 \cdot 1$