Logic and Set Theory Relationship

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The following are translations of logic ideas into set theory ideas.

The symbols p, q, and r denote arbitrary statements, and **T** and **F** denote "True" and "False" respectively. Let A, B, and C denote sets and let U denote a "universal set" (i.e. U is the set theoretic version of **T**). The idea below is to translate a statement p into the set theory statement $x \in A$, etc.

Name	**Logic**	**Sets**
Double Negation Law	$\neg(\neg \mathrm{p}) \equiv \mathrm{p}$	$(A^{\mathsf{C}})^{\mathsf{C}} = A$
Negation Laws	$\mathrm{p} \vee (\neg \mathrm{p}) \equiv \mathbf{T}$	$A \cup A^{\mathsf{C}} = U$
	$\mathrm{p} \wedge (\neg \mathrm{p}) \equiv \mathbf{F}$	$A \cap A^{\mathsf{C}} = \emptyset$
Idempotence Laws	$\mathrm{p} \vee \mathrm{p} \equiv \mathrm{p}$	$A \cup A = A$
	$\mathrm{p} \wedge \mathrm{p} \equiv \mathrm{p}$	$A \cap A = A$
Identity Laws	$\mathrm{p} \wedge \mathbf{T} \equiv \mathrm{p}$	$A \cap U = A$
	$\mathrm{p} \vee \mathbf{F} \equiv \mathrm{p}$	$A \cup \emptyset = A$
Domination Laws	$\mathrm{p} \vee \mathbf{T} \equiv \mathbf{T}$	$A \cup U = U$
	$\mathrm{p} \wedge \mathbf{F} \equiv \mathbf{F}$	$A \cap \emptyset = \emptyset$
Commutative Laws	$\mathrm{p} \vee \mathrm{q} \equiv \mathrm{q} \vee \mathrm{p}$	$A \cup B = B \cup A$
	$\mathrm{p} \wedge \mathrm{q} \equiv \mathrm{q} \wedge \mathrm{p}$	$A \cap B = B \cap A$
Associative Laws	$\mathrm{p} \vee (\mathrm{q} \vee \mathrm{r}) \equiv (\mathrm{p} \vee \mathrm{q}) \vee \mathrm{r}$	$A \cup (B \cup C) = (A \cup B) \cup C$
	$\mathrm{p} \wedge (\mathrm{q} \wedge \mathrm{r}) \equiv (\mathrm{p} \wedge \mathrm{q}) \wedge \mathrm{r}$	$A \cap (B \cap C) = (A \cap B) \cap C$
Distributive Laws	$\mathrm{p} \vee (\mathrm{q} \wedge \mathrm{r}) \equiv (\mathrm{p} \vee \mathrm{q}) \wedge (\mathrm{p} \vee \mathrm{r})$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
	$\mathrm{p} \wedge (\mathrm{q} \vee \mathrm{r}) \equiv (\mathrm{p} \wedge \mathrm{q}) \vee (\mathrm{p} \wedge \mathrm{r})$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
De Morgan's Laws	$\neg(\mathrm{p} \vee \mathrm{q}) \equiv (\neg \mathrm{p}) \wedge (\neg \mathrm{q})$	$(A \cup B)^{\mathsf{C}} = A^{\mathsf{C}} \cap B^{\mathsf{C}}$
	$\neg(\mathrm{p} \wedge \mathrm{q}) \equiv (\neg \mathrm{p}) \vee (\neg \mathrm{q})$	$(A \cap B)^{\mathsf{C}} = A^{\mathsf{C}} \cup B^{\mathsf{C}}$
$\oplus$-Definition	$\mathrm{p} \oplus \mathrm{q} \equiv (\mathrm{p} \vee \mathrm{q}) \wedge (\neg(\mathrm{p} \wedge \mathrm{q}))$	$A \triangle B = (A \cup B) \cap (A \cap B)^{\mathsf{C}}$
Material Implication	$\mathrm{p} \to \mathrm{q} \equiv (\neg \mathrm{p}) \vee \mathrm{q}$	$A \subseteq B$ if and only if $U = A^{\mathsf{C}} \cup B$
Contraposition	$\mathrm{p} \to \mathrm{q} \equiv (\neg \mathrm{q}) \to (\neg \mathrm{p})$	$A \subseteq B$ if and only if $B^{\mathsf{C}} \subseteq A^{\mathsf{C}}$
Biconditional Expansion	$\mathrm{p} \leftrightarrow \mathrm{q} \equiv (\mathrm{p} \to \mathrm{q}) \wedge (\mathrm{q} \to \mathrm{p})$	$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

Modus Ponens	$\mathrm{p} \to \mathrm{q}, \mathrm{p} \;\; \therefore \mathrm{q}$	If $A \subseteq B$ and $x \in A$, then $x \in B$.
Modus Tollens	$\mathrm{p} \to \mathrm{q}, \neg \mathrm{q} \;\; \therefore \neg \mathrm{p}$	If $A \subseteq B$ and $x \notin B$, then $x \notin A$.
Disjunctive Syllogism	$\mathrm{p} \vee \mathrm{q}, \neg \mathrm{p} \;\; \therefore \mathrm{q}$	If $x \in A \cup B$ and $x \notin A$, then $x \in B$.
Hypothetical Syllogism	$\mathrm{p} \to \mathrm{q}, \mathrm{q} \to \mathrm{r} \;\; \therefore \mathrm{p} \to \mathrm{r}$	If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.
Reductio Ad Absurdum	$\mathrm{p} \to (\mathrm{q} \wedge (\neg \mathrm{q})) \;\; \therefore \neg \mathrm{p}$	If $A \subseteq \emptyset$, then $A = \emptyset$.
Conjunctive Addition	$\mathrm{p}, \mathrm{q} \;\; \therefore \mathrm{p} \wedge \mathrm{q}$	If $x \in A$ and $x \in B$, then $x \in A \cap B$.
Conjunctive Simplification	$\mathrm{p} \wedge \mathrm{q} \;\; \therefore \mathrm{p}$	$A \cap B \subseteq A$
Disjunctive Addition	$\mathrm{p} \;\; \therefore \mathrm{p} \vee \mathrm{q}$	$A \subseteq A \cup B$