Homework 9

Instructions: Complete each of the following on separate, stapled sheets of paper.

1. Rewrite the expressions below into a single power series by reindexing.

   (a) $\sum_{n=1}^{\infty} nc_n x^{n-1} - \sum_{n=0}^{\infty} c_n x^n$

   (b) $\sum_{n=1}^{\infty} nc_n x^{n-1} + 3\sum_{n=0}^{\infty} c_n x^{n+2}$

   (c) $\sum_{n=1}^{\infty} 2nc_n x^{n-1} + \sum_{n=0}^{\infty} 6c_n x^{n+1}$

   (d) $\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} + \sum_{n=0}^{\infty} c_n x^{n+2}$

   (e) $\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} - 2\sum_{n=1}^{\infty} nc_n x^n + \sum_{n=0}^{\infty} c_n x^n$

   (f) $\sum_{n=2}^{\infty} n(n-1)c_n x^n + 2\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} + 3\sum_{n=1}^{\infty} nc_n x^n$

2. Verify that the given power series satisfies the indicated ODE by direct substitution.

   (a) $y(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}; \qquad y' + 2xy = 0$

   (b) $y(x) = \sum_{n=0}^{\infty} (-1)^n x^{2n}; \qquad (1+x^2)y' + 2xy = 0$

   (c) $y(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n; \qquad (x+1)y'' + y' = 0.$

3. Use Maclaurin series to solve the following ODEs; give the recursion, but don't solve it explicitly (yet).

   (a) $y'' + y = 0$

   (b) $y'' - y = 0$

   (c) $y'' - y' = 0$

   (d) $y'' + 2y' = 0$

   (e) $y'' + xy = 0$

   (f) $y'' + 2xy + 2y = 0$