Homework 7

Instructions: Complete each of the following on separate, stapled sheets of paper.

1. Give general solutions to the second-order homogeneous linear ODEs below, given that y_1 is a solution (verify).

   (a) $xy'' + y' = 0; \quad y_1 = \ln(x)$
   (b) $4x^2y'' + y = 0; \quad y_1 = \sqrt{x}\ln(x)$
   (c) $x^2y'' - 3xy' + 5y = 0; \quad y_1 = x^2\cos(\ln(x))$
   (d) $(1 - x^2)y'' + 2xy' = 0; \quad y_1 = 1$

2. Solve the following nonhomogeneous linear ODEs via reduction of order (**Hint**: Solve the associated homogeneous equation, and then pick one of those solutions to use for y_1 when solving the nonhomogeneous equation).

   (a) $y'' + y' = 1$
   (b) $y'' - 4y' + 3y = x$

3. Write each ODE below in the form $L(y) = g(x)$ for L a linear differential operator; factor L if possible.

   (a) $9y'' - 4y = \sin(x)$
   (b) $2y'' - 3y' - 2y = 1$
   (c) $y''' + 4y'' + 3y' = x^2\cos(x) - 3x$
   (d) $y^{(4)} - 8y'' + 16y = (x^3 - 2x)e^{4x}$

4. Verify the given differential operator is an annihilator of the indicated function.

   (a) $D^4; \quad y = 10x^3 - 2x$
   (b) $2D - 1; \quad y = 4e^{\frac{x}{2}}$
   (c) $(D - 2)(D + 5); \quad y = e^2x + 3e^{-5x}$
   (d) $D^2 + 64; \quad y = 2\cos(8x) - 5\sin(8x)$

5. Find a linear differential operator that annihilates the given function.

   (a) $x^3(1 - 5x)$
   (b) $1 + 7e^{2x}$
   (c) $1 + \sin(x)$
   (d) $8x - \sin(x) + 10\cos(5x)$
   (e) $e^{-x} + 2xe^x - x^2e^x$
   (f) $3 + e^x\cos(2x)$

6. Find linearly independent functions which are annhilated by the given differential operator.

   (a) $D^2 - 9D - 36$
   (b) $D^2 + 5$
   (c) $D^3 - 10D^2 + 25D$
   (d) $D^2(D - 5)(D - 7)$

7. Use the method of undetermined coefficients to solve the following ODEs.

   (a) $y'' - 9y = 54$
   (b) $y'' - 2y' + y = x^3 + 4x$
   (c) $y'' + 4y = 4\cos(x) + 3\sin(x) - 8$
   (d) $y'' + y' + \frac{1}{4}y = e^x(\sin(3x) - \cos(3x))$
   (e) $y'' + y' + y = x\sin(x)$
   (f) $2y''' - 3y'' - 3y' + 2y = (e^x + e^{-x})^2$

8. Solve the following IVPs.

   (a) $y'' + y = 8\cos(2x) - 4\sin(x); \quad y(\frac{\pi}{2}) = 1, \; y'(\frac{\pi}{2}) = 0$
   (b) $y^{(4)} - y^{(3)} = x + e^x; \quad y(0) = 0, \; y'(0) = 0, \; y''(0) = 0, \; y'''(0) = 0$