Homework 4

Instructions: Complete each of the following on separate, stapled sheets of paper.

1. Compute the differential df for each of the following functions.

   (a) $f(x, y) = \cos(xy)\sin(y) + \cos(x)$

   (b) $f(x, y) = e^{xy^3}$

2. Determine whether the given differential equation is exact. If it is exact, solve it.

   (a) $x\dfrac{dy}{dx} = 2xe^x - y + 6x^2$

   (b) $(5y - 2x)\dfrac{dy}{dx} = 2y$

   (c) $\left(1 + \ln(x) + \dfrac{y}{x}\right)\, dx = (1 - \ln(x))\, dy$

   (d) $(y\ln(y) - e^{-xy})\, dx + \left(\dfrac{1}{y} + x\ln(y)\right)\, dy = 0$

3. Solve the following nonexact ODEs by finding the appropriate integrating factors.

   (a) $(2y^2 + 3x)\, dx + 2xy\, dy = 0$

   (b) $(10 - 6y + e^{-3x})\, dx - 2\, dy = 0$

4. Compute the general solution to the following linear ODEs.

   (a) $\dfrac{dy}{dx} + y = e^{3x}$

   (b) $x\dfrac{dy}{dx} - y = x^2\sin(x)$

   (c) $\dfrac{dr}{d\theta} + r\sec(\theta) = \cos(\theta)$

   (d) $(x+2)^2\dfrac{dy}{dx} = 5 - 8y - 4xy$

5. Solve the following ODEs (first verify they are Bernoulli equations).

   (a) $\dfrac{dy}{dx} - y = e^x y^2$

   (b) $t^2\, dy + (y^2 - ty)\, dt = 0$

   (c) $x\dfrac{dy}{dx} + y = \dfrac{1}{y^2}$

   (d) $x\, dy - y(1 + x + xy)\, dx = 0$

6. Solve the following ODE (reduce to a separable equation via a nice substitution).

   (a) $\dfrac{dy}{dx} = (x + y + 1)^2$

   (b) $(x + y)\, dy = (1 - x - y)\, dx$

   (c) $\dfrac{dy}{dx} = \sin(x + y)$

   (d) $dy - \tan^2(x + 3y - 5)\, dx = 0$

7. Solve the following ODEs (first verify that they are homogeneous).

   (a) $(y^2 + xy)\, dx - x^2 dy = 0$

   (b) $\dfrac{dy}{dx} = \dfrac{y - x}{x + y}$

   (c) $(3x + y)\, dx = (x + 3y)\, dy$

   (d) $x\dfrac{dy}{dx} = y + \sqrt{x^2 - y^2}$ (assume $x > 0$)

8. Solve the following IVPs.

   (a) $xy^2\dfrac{dy}{dx} = y^3 - x^3, \quad y(1) = 2$

   (b) $(x^2 + y^2 - 5)\, dx = (y + xy)\, dy, \quad y(4) = 0$

   (c) $(x + y\exp(\frac{y}{x}))\, dx - x\exp(\frac{y}{x})\, dy = 0, \quad y(1) = 2$

   (d) $x(x+1)\dfrac{dy}{dx} + xy = e^x, \quad y(e) = 1$

   (e) $y^{\frac{1}{2}}\dfrac{dy}{dx} + y^{\frac{3}{2}} = 1, \quad y(0) = 4$

   (f) $x^2\dfrac{dy}{dx} - 2xy = 3y^4, \quad y(1) = \frac{1}{2}$

   (g) $(3x + 2y + 2)\, dy - (3x + 2y)\, dx = 0, \quad y(-1) = -1$

   (h) $(x + y)^2\, dx + (2xy + x^2 - 1)\, dy = 0, \quad y(1) = 1$

   (i) $\dfrac{dy}{dx} = \cos(x + y), \quad y(0) = \frac{\pi}{4}$

   (j) $y' + \tan(x)y = \cos^2(x), \quad y(0) = -1$

9. Study for Midterm 1.