Homework 1: Logic and Set Theory

Due 15 February 2019

Instructions: Legibly complete each of the following on *separate stapled* sheets of paper.

1. Using the dictionary below, translate the following English statements into the formal propositional language.

   A : Amy can pitch a tent.
   B : Amy can go camping.
   C : Amy likes to go fishing.
   D : Amy knows how to build a campfire.

   (a) If Amy can pitch a tent, then she can go camping.
   (b) If Amy likes to go fishing, then she knows how to build a campfire but cannot go camping.
   (c) If Amy does not know how to build a campfire but can pitch a tent, then either Amy does not like to fish or she can go camping.

2. Using the dictionary from Question 1, translate the following formal statements into English.
   (a) $A \implies (B \vee (\neg C))$
   (b) $A \iff D$
   (c) $(A \wedge (\neg C)) \vee (C \wedge (\neg B))$

3. Create a truth table for each of the following propositional statements.
   (a) $(P \implies Q) \implies (Q \implies P)$
   (b) $(Q \iff (\neg P)) \vee P$
   (c) $((P \implies Q) \wedge P) \implies Q$
   (d) $(P \oplus Q) \wedge (P \iff Q)$
   (e) $((P \implies Q) \wedge (Q \implies R)) \iff (P \implies R)$
   (f) $((P \oplus Q) \oplus R) \iff (P \oplus (Q \oplus R))$

4. Compute the disjunctive normal form (DNF) of each statement from Question 3.

5. Compute the conjunctive normal form (CNF) of each statement from Question 3.

6. For each statement below, find a model in which the statement is False.
   (a) $(\forall x\ P(x)) \implies (\exists x\ P(x))$
   (b) $(\exists x\ P(x)) \implies (\forall x\ P(x))$
   (c) $[\forall x\ \exists y\ P(x,y)] \implies [\exists y\ \forall x\ P(x,y)]$

7. Let A, B, and C be sets. Prove each of the following.
   (a) $A \cup B = B \cup A$
   (b) $A \cap B = B \cap A$
   (c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (d) $A - (B \cup C) = (A - B) \cap (A - C)$
   (e) $A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$
   (f) $A - (A - B) = A \cap B$

8. Let A and B be sets. Prove each of the following.
   (a) $A \cap B \subseteq A$
   (b) $A \subseteq A \cup B$
   (c) If $A \subseteq B$, then $\text{pow}(A) \subseteq \text{pow}(B)$.