Homework 1

Instructions: Complete each problem on separate, stapled sheets of paper. *Show all necessary work.*

1. State whether each expression is meaningful or meaningless. Explain.

   (a) $(\vec{u} \cdot \vec{v}) \cdot \vec{w}$

   (b) $(\vec{u} \cdot \vec{v})\vec{w}$

   (c) $|\vec{u}|(\vec{v} \cdot \vec{w})$

   (d) $\vec{u} \cdot (\vec{v} + \vec{w})$

   (e) $\vec{u} \cdot \vec{v} + \vec{w}$

   (f) $|\vec{u}| \cdot (\vec{v} + \vec{w})$

   (g) $\vec{u} \cdot (\vec{v} \times \vec{w})$

   (h) $\vec{u} \times (\vec{v} \cdot \vec{w})$

   (i) $\vec{u} \times (\vec{v} \times \vec{w})$

   (j) $\vec{u} \cdot (\vec{v} \cdot \vec{w})$

   (k) $(\vec{u} \cdot \vec{v}) \times (\vec{w} \cdot \vec{x})$

   (l) $(\vec{u} \times \vec{v}) \cdot (\vec{w} \times \vec{x})$

2. Show the equation $x^2 + y^2 + z^2 - 2x - 4y + 8z = 15$ represents a sphere, and find its center and radius.
3. Compute the angle between vectors $\vec{u} = -8\vec{i} + 12\vec{j} + 4\vec{k}$ and $\vec{v} = 6\vec{i} - 9\vec{j} - 3\vec{k}$. Are they orthogonal? Parallel?
4. Find the direction cosines and direction angles of the vector $\langle 2, 1, 2 \rangle$.
5. Find the scalar and vector projections of $\vec{u} = \langle -1, 4, 8 \rangle$ onto $\vec{v} = \langle 12, 1, 2 \rangle$.
6. Find the area of the quadrilateral with vertices $A = (-3, 0)$, $B = (-1, 3)$, $C = (5, 2)$, and $D = (3, -1)$.
7. Compute the vector equation for the line through $(2, 1, 0)$ and perpendicular to both $\vec{i} + \vec{j}$ and $\vec{j} + \vec{k}$.
8. Compute an equation of the plane...

   (a) containing line $x = 1 + t, y = 2 - t, z = 4 - 3t$ and parallel to $5x + 2y + z = 1$.

   (b) through $(3, 1, 4)$ and containing the line of intersection of the planes $x + 2y + 3z = 1$ and $2x - y - z = 2$.
9. Prove $|\vec{u} + \vec{v}|^2 + |\vec{u} - \vec{v}|^2 = 2|\vec{u}|^2 + 2|\vec{v}|^2$ for all vectors $\vec{u}$ and $\vec{v}$.
10. Let $\vec{u}$ and $\vec{v}$ be vectors with $\vec{w} \coloneqq |\vec{v}|\vec{u} + |\vec{u}|\vec{v} \neq \vec{0}$. Show that $\vec{w}$ bisects the angle between $\vec{u}$ and $\vec{v}$.