

1. A machine has been designed to toss a coin automatically and keep track of the number of heads. After 1000 tosses, it has 550 heads. Express the chance error both in absolute terms and as a percentage of the number of tosses.
2. After 1000000 tosses, the machine in the above exercise has 501000 heads. Express the chance error in the same two ways.
3. A coin is tossed 100 times, landing heads 53 times. However, the last seven tosses were all heads. True or false: the chance that the next toss will be heads is somewhat less than 50%.
4. (a) A coin is tossed, and you win a dollar if there are more than 60% heads. Which is better: 10 tosses or 100? Explain.  
 (b) As above, but you the dollar if there are more than 40% heads.  
 (c) As above, but you win the dollar if there are between 40% and 60% heads.  
 (d) As above, but you win the dollar if there are exactly 50% heads.
5. With a Nevada roulette wheel, there are 18 chances in 38 that the ball will land in a red pocket. A wheel is going to be spun many times. There are two choices:
  - (a) 38 spins, and you win a dollar if the ball lands in a red pocket 20 or more times.
  - (b) 76 spins, and you win a dollar if the ball lands in a red pocket 40 or more times.

Which is better? Or are they the same? Explain.
6. A box contains 20% red marbles and 80% blue marbles. A thousand marbles are drawn at random with replacement. One of the following statements is true. Which one, and why?
  - (a) Exactly 200 marbles are going to be red.
  - (b) About 200 marbles are going to be red, give or take a dozen or so.
7. Same as above, but the draws are now made at random without replacment and there are 50000 marbles in the box.
8. One hundred tickets will be drawn at random with replacment from one of two boxes. On each draw, you will be paid the amount shown on the ticket, in dollars. (If a negative number is drawn, that amount will be taken away from you.) Which box is better? Or are they the same?
  - (a) A box with four tickets: two labeled "1" and two labeled "-1".
  - (b) A box with two tickets: one labeled "1" and one labeled "-1".

9. One hundred draws are made at random with replacement from a box with two tickets: one labeled “1” and one labeled “2”. Forty-seven draws turn out to be “1” and 53 are “2”. How much is the sum?
10. One hundred draws are made at random with replacement from a box with two tickets: one labeled “1” and one labeled “2”.
  - (a) How small can the sum be? How large?
  - (b) How many times do you expect the ticket “1” to show up? The ticket “2”?
  - (c) About how much do you expect the sum to be?
11. One hundred draws are made at random with replacement from a box with three tickets: one labeled “1”, one labeled “2”, and one labeled “9”.
  - (a) How small can the sum be? How large?
  - (b) About how much do you expect the sum to be?
12. One hundred draws are made at random with replacement from one of the following boxes. Your job is to guess what the sum will be, and you win \$1 if you are right to within 10. In each case, what would you guess? Which box is best? Worst?
  - (a) A box with two tickets: one labeled “1” and one labeled “9”.
  - (b) A box with two tickets: one labeled “4” and one labeled “6”.
  - (c) A box with two tickets: both labeled “5”.
13. One ticket will be drawn at random from a box with ten tickets labeled “1” through “10”. What is the chance it will be “1”? That it will be “3” or less? “4” or more?
14. Fifty draws will be made at random with replacement from one of the two boxes shown below. On each draw you will be paid in dollars the amount shown on the ticket; if a negative number is drawn, that amount will be taken away from you. Which box is better? Or are they the same? Explain.
  - (a) A box with two tickets: one labeled “-1” and one labeled “2”
  - (b) A box with three tickets: two labeled “-1” and one labeled “2”
15. Consider the following three situations.
  - (a) A box contains one ticket marked “0” and nine marked “1”. A ticket is drawn at random. If it shows “1” you win a panda bear.
  - (b) A box contains ten tickets marked “0” and ninety marked “1”. A ticket is drawn at random. If it shows “1” you win a panda bear.

- (c) A box contains one ticket marked “0” and nine marked “1”. Ten draws are made at random with replacement. If the sum of the draws equals 10, you win the panda.

Assume you want the panda. Which is better – (a) or (b)? Or are they the same? What about (a) and (c)?

16. A gambler is going to play roulette 25 times, putting a dollar on a *split* each time. (A split is two adjacent numbers like 11 and 12.) If either number shows up, the gambler takes the dollar back, together with winnings of \$17. If neither number comes up, he loses the dollar. So a split pays 17 to 1, and there are 2 chances in 38 to win. The gambler's net gain in the 25 plays is like the sum of 25 draws made from one of the following boxes. Which one, and why?
- (a) A box with 38 tickets: one labeled “0”, one labeled “00”, and 36 labeled “1” through “36”.
- (b) A box with 36 tickets: two labeled “17”, 34 labeled “-1”.
- (c) A box with 38 tickets: two labeled “17”, 36 labeled “-1”.
17. In one version of chuck-a-luck, 3 dice are rolled out of a cage. You can bet that all 3 show six. The house pays 36 to 1, and the bettor has 1 chance in 216 to win. Suppose you make this bet 10 times, staking \$1 each time. Your net gain is like the sum of hspace2cm draws made at random with replacement from the box hspace2cm. Fill in the blanks.
18. A box contains 10000 tickets: 4000 labeled “0” and 6000 labeled “1” / An 10000 draws will be made at random with replacement from this box. Which of the following best describes the situation, and why?
- (a) The number of 1's will be 6000 exactly.
- (b) The number of 1's is very likely to equal 6000, but there is also some small chance that it will not be equal to 6000.
- (c) The number of 1's is likely to be different from 6000, but the difference is likely to be small relative to 10000.
19. Repeat the above exercise for 10000 draws made at random without replacement.
20. A gambler loses ten times running at roulette. He decides to continue playing because he is due for a win, by the law of averages. A bystander advises him to quit, on the grounds that his luck is cold. Whos is right? Or are both of them wrong?
21. (a) A die will be rolled some number of times, and you win \$1 if it shows an ace more than 20% of the time. What is better: 60 rolls or 600 rolls? Explain.

- (b) As above, but you win the dollar if the percentage of aces is more than 15%.
  - (c) As above, but you win the dollar if the percentage of aces is between 15% and 20%.
  - (d) As above, but you win the dollar if the percentage of aces is exactly  $16\frac{2}{3}\%$ .
22. True or false: if a coin is tossed 100 times, it is not likely that the number of heads will be exactly 50, but it is likely that the percentage of heads will be exactly 50%.
23. According to genetic theory, there is very close to an even chance that both children in a two-child family will be of the same sex. Here are two possibilities:
- (a) 15 couples have two children each. In 10 or more of these families, it will turn out that both children are of the same sex.
  - (b) 30 couples have two children each. In 20 or more of these families, it will turn out that both children are of the same sex.

Which possibility is more likely, and why?

24. A quiz has 25 multiple choice questions. Each question has 5 possible answers, one of which is correct. A correct answer is worth 4 points, but a point is taken off for each incorrect answer. A student answers all the questions by guessing at random. The score will be like the sum of hspace2cm draws from the box hspace2cm. Explain your answers.
25. A gambler will play roulette 50 times, betting a dollar on four *joining* numbers each time (like 23, 24, 26, 27). If one of these four numbers comes up, she gets the dollar back, together with winnings of \$8. If any other number comes up, she loses the dollar. So this bet pays 8 to 1, and there are 4 chances in 38 of winning. Her net gain in 50 plays is like the sum of hspace2cm draws from the box hspace2cm. Fill in the blanks; explain.
26. A box contains red marbles and blue marbles; there are more red marbles than blue ones. Marbles are drawn one at a time from the box, at random with replacement. You win a dollar if a red marble is drawn more often than a blue one. There are two choices:
- 100 draws are made from the box.
  - 200 draws are made from the box.

Choose one of the four options below. Explain your answer.

- (a) The first choice gives a better chance of winning.
- (b) The second choice gives a better chance of winning.

- (c) Both choices give the same chance of winning.
  - (d) Can't tell without more information.
27. Two hundred draws will be made at random with replacement from the box with 7 tickets labeled “-3” through “3”.
- (a) If the sum of the 200 numbers drawn is 30, what is their average?
  - (b) If the sum of the 200 numbers drawn is -20, what is their average?
  - (c) In general, how can you figure out the average of the 200 draws, if you are told their sum?
  - (d) There are two alternatives:
    - i. winning \$1 if the sum of the 200 numbers is between -5 and 5.
    - ii. winning \$1 if the average of the 200 numbers drawn is between -0.025 and 0.025

Which is better, or are they the same? Explain.