1. Find the number of different ways of arranging one R and three G's in a row. Write out all the patterns.

2. Find the number of different ways of arranging two R's and two G's in a row. Write out all the patterns.

3. A box contains one red ball and five green ones. Four draws are made at random with replacement from the box. Find the chance that –

   (a) a red ball is never drawn.

   (b) a red ball appears exactly once.

   (c) a red ball appears exactly twice.

   (d) a red ball appears exactly three times.

   (e) a red ball appears on all draws.

   (f) a red ball appears at least twice.

4. A die is rolled four times. Find the chance that –

   (a) an ace never appears.

   (b) an ace appears exactly once.

   (c) am ace appears exactly twice.

5. A coin is tossed 10 times. Find the chance of getting exactly 5 heads. Find the chance of getting between 4 and 6 heads inclusive.

6. It is claimed that a vitamin supplement helps kangaroos learn to run a special maze with high walls. TO test whether this is true, 20 kangaroos are divided up into 10 pairs. In each pair, one kangaroo is selected at random to receive the vitamin supplement; the other is fed a normal diet. The kangaroos are then timed as they learn to run the maze. In 7 out of 10 pairs, the treated kangaroo learns to run the maze more quickly than its untreated partner. If in fact the vitamin supplement has no effect, so that each animal of the pair is qually likely to be the quicker, what is the probability that 7 or more of the treated animals would learn the maze more quickly than their untreadted partners, just by chance?

7. A die will be rolled 6 times. What is the chance of obtaining exactly one ace?

8. A die will be roled 10 times. What is the chance that it never lands six?

9. Of families with 4 children, what proportion have more girls than boys? You may assume that the sex of the child is determined as if by drawing at random with replacement from a box with two tickets: one labeled "M" and one labeled "F".

10. A box contains 8 red marbles and 3 green ones. Six draws are made at random without replacement. True or false: the chance that the three green marbles are drawn equals $\binom{6}{3}(\frac{8}{11})^3(\frac{3}{11})^3$. Explain briefly.

11. There are 8 people in a club. One person makes up a list of all possible committees with 2 members. Another person makes up a list of all possible committees with 5 members. True or false: the second list is longer than the first.

12. There are 8 people in a club. One person makes up a list of all possible committees with 2 members. Another person makes up a list of all possible committees with 6 members. True or false: the second list is longer than the first.

13. A box contains one red marble and nine green ones. Five draws are made at random with replacement. The chance that exactly two draws will be red is $10 \cdot (\frac{1}{10})^2(\frac{9}{10})^3$. Is the addition rule used in deriving this formula? Answer yes or no, and explain carefully.

14. A coin will be tossed 10 times. Find the chance that there will be exactly 2 heads among the first 5 tosses, and eactly 4 heads among the last 5 tosses.

15. A deck of cards is shuffled. What is the chance that –

    (a) the top card is the king of spades and the bottom card is the queen of spades?

    (b) the top card is the king of spades and the bottom card is the king of spades?

    (c) the top card is the king of spades or the bottom card is the king of spades?

    (d) the top card is the king of spades or the bottom card is the queen of spades?

    (e) of the top and bottom cards, one is the king of spades and the other is the queen of spades?

16. A box contains 3 red tickets and 2 green ones. Five draws will be made at random. You win \$1 if 3 of the draws are red and 2 are green. Would you prefer the draws to be made with or without replacement? Why?

17. It is now generally accepted that cigarette smoking causes heart disease, lung cancer, and many other diseases. However, in the 1950s, this idea was controversial. There was a strong association between smoking and ill-health, but association is not causation. RA Fisher advanced the "constitutional hypothesis:" there is some genetic factor that disposes you both to smoke and to die.

To refute Fisher's idea, the epidemiologists used twin studies. They identified sets of smoking-discordant monozygotic twin pairs. ("Monozygotic" twins come from one egg and have identical genetic make-up; "smoking-discordant" means one twin smokes, the other doesn't.) Now there is a race. Which twin dies first, the smoker or the non-smoker? Data from a Finnish study are shown below:

	Smokers	Non-smokers
All causes	17	5
Coronary heart disease	9	0
Lung cancer	2	0

According to the first line of the table, there were 22 smoking-discordant monozygotic twin pairs where at least one twin of the pair died. In 17 cases, the smoker died first; in 5 cases, the non-smoker died first. According to the second line, there were 9 pairs where at least one twin died of coronary heart disease; in all 9 cases, the smoker died first. According to the last line, there were two pairs where at least one twin died of lung cancer , and in both pairs the smoker won the race to death. (Lung cancer is a rare disease, even among smokers.)

For the first three questions, suppose that each twin in the pair is equally likely to die first, so the number of pairs in which the smoker dies first is like the number of heads in coin-tossing.

(a) On this basis, what is the chance of having 17 or more pairs out of 22 where the smoker dies first?

(b) Repeat the test for the 9 deaths from coronary heart disease.

(c) Repeat the test for the two deaths from lung cance.

(d) Can the difference between the death rates for smoking and non-smoking twins be explained by

   i. chance?

   ii. genetics?

   iii. health effects of smoking?