

# Math 324 Quiz 7

20 June 2016

Name: Answer Key

1. Find a nontrivial solution to  $4x^2y'' + y = 0$ .

$$y'' + \frac{1}{4x^2}y = 0$$

0 is not ordinary

$$x^2 0 = \sum a_k x^k \rightarrow a_0 = 0$$

$$x^2 \left( \frac{1}{4x^2} \right) = \frac{1}{4} = \sum b_k x^k \rightarrow b_0 = \frac{1}{4}$$

0 is regular singular

$$q(r) = r(r-1) + \frac{1}{4} = r^2 - r + \frac{1}{4} = \left(r - \frac{1}{2}\right)\left(r - \frac{1}{2}\right)$$

$$\text{roots} = \frac{1}{2}, \frac{1}{2}$$

$$\text{guess } y = \sum_{k=0}^{\infty} c_k x^{k+r}$$

$$0 = \sum_{k=0}^{\infty} 4(k+r)(k+r-1)c_k x^{k+r} + \sum_{k=0}^{\infty} c_k x^{k+r}$$

$$= (4r(r-1) + 1)c_0 x^r + \sum_{k=1}^{\infty} [4(k+r)(k+r-1) + 1]c_k x^{k+r}$$

$$\text{so } r = \frac{1}{2} \quad \text{and} \quad c_k = 0 \quad \text{for } k \geq 1,$$

$$\text{hence } y = x^{1/2}$$

Since the equation is Cauchy-Euler,  
 $y = c_1 x^{1/2} + c_2 x^{1/2} \ln(x)$