Math 324 Quiz 4

8 June 2016

Name: Answer Key

1. Find the general solution of $y'' + 3y' + 2y = \frac{1}{1+e^x}$.

$r^2 + 3r + 2 = 0$

$(r+2)(r+1)$

roots $= -1, -2$

$y_c = c_1 e^{-x} + c_2 e^{-2x}$

$W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -2e^{-3x} + e^{-3x} = -e^{-3x}$

$W_1 = \begin{vmatrix} 0 & e^{-2x} \\ \frac{1}{1+e^x} & -2e^{-2x} \end{vmatrix} = -\frac{e^{-2x}}{1+e^x}$

$W_2 = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & \frac{1}{1+e^x} \end{vmatrix} = \frac{e^{-x}}{1+e^x}$

$v = e^x + 1$

$dv = e^x dx$

$u_1' = \frac{e^x}{1+e^x}$, $u_1 = \int \frac{e^x}{1+e^x} dx = \int \frac{1}{v} dv = \ln(1+e^x)$

$u_2' = \frac{-e^{2x}}{1+e^x}$, $u_2 = -\int \frac{e^{2x}}{1+e^x} dx = -\int \frac{v-1}{v} dv$

$= \ln(1+e^x) - 1 - e^x$

$y_p = \ln(1+e^x)e^{-x} + (\ln(1+e^x) - 1 - e^x)e^{-2x}$

$y = y_p + y_c$

2. Find the general solution of $y'' - 9y = \frac{9x}{e^{3x}}$.

$r^2 - 9 = 0$

$(r+3)(r-3)$

roots $= \pm 3$

$y_c = c_1 e^{3x} + c_2 e^{-3x}$

$W = \begin{vmatrix} e^{3x} & e^{-3x} \\ 3e^{3x} & -3e^{-3x} \end{vmatrix} = -3 - 3 = -6$

$W_1 = \begin{vmatrix} 0 & e^{-3x} \\ 9xe^{-3x} & -3e^{-3x} \end{vmatrix} = -9xe^{-6x}$

$W_2 = \begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & 9xe^{-3x} \end{vmatrix} = 9x$

$u_1' = \frac{3}{2} x e^{-6x}$

$u_2' = -\frac{3}{2} x$, $u_2 = -\frac{3}{4} x^2$,

$u_1 = \frac{3}{2} \int x e^{-6x} dx = \frac{3}{2} \left(-\frac{1}{6} x e^{-6x} - \frac{1}{36} e^{-6x} \right)$

$r = x \longrightarrow dr = dx$

$ds = e^{-6x} dx \longrightarrow s = -\frac{1}{6} e^{-6x}$

$y_p = \frac{3}{2} \left(-\frac{1}{6} x e^{-6x} - \frac{1}{36} e^{-6x} \right) e^{3x} - \frac{3}{4} x^2 e^{-3x}$

$y = y_p + y_c$

3. (Bonus) Can you use the annihilator method to solve problem 2?

yes.