

Math 220 Section 6 Exam 2

28 October 2015

Name: Key

1. Let $f(x) = \ln\left(\frac{x}{x+1}\right)$. Find the horizontal and vertical asymptotes of $f(x)$.

$$= \ln(x) + \ln(x+1)$$

so domain is $(0, \infty)$

$$\lim_{x \rightarrow \infty} \ln\left(\frac{x}{x+1}\right) = \ln\left(\lim_{x \rightarrow \infty} \frac{x}{x+1}\right) = \ln\left(\lim_{x \rightarrow \infty} \frac{1}{1+\frac{1}{x}}\right) = \ln(1) = 0$$

$$\lim_{x \rightarrow 0^+} \ln\left(\frac{x}{x+1}\right) = \ln\left(\lim_{x \rightarrow 0^+} \frac{x}{x+1}\right) = \ln(0) = -\infty$$

so $x=0$ is VA and $y=0$ is HA.

2. Let $g(z) = 4z^2 - 4z^4$. What are the absolute maximum and absolute minimum of $g(z)$ on the interval $[0, 2]$?

$$g'(z) = 8z - 16z^3 = 8z(1 - 2z^2)$$

$$z = \pm \frac{1}{\sqrt{2}}$$

0	0	(0,0)
$\frac{1}{\sqrt{2}}$	$2 - 1 = 1$	$\left(\frac{1}{\sqrt{2}}, 1\right)$ max
2	$16 - 64 = -48$	$(2, -48)$ min

$\left(\frac{1}{\sqrt{2}}, 1\right)$ max
 $(2, -48)$ min

3. Find the slope of the tangent line to the curve $y^3 + xy - y = 8x^4$ at the point $(1, 2)$.

$$3y^2 y' + xy' + y - y' = 32x^3$$

$$3(4)y' + y' + 2 - y' = 32$$

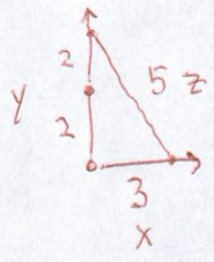
$$12y' = 32$$

$$y' = \frac{32}{12}$$

34

4. A train leaves the station at 9:00 a.m. heading north at 2 kilometers per hour. Another train leaves the station at 10:00 a.m. heading east at 3 kilometers per hour. How fast is the distance between the two trains changing at 11:00 a.m.?

12



$$x^2 + y^2 = z^2$$

$$3(3) + (4)(2) = 5 \frac{dz}{dt}$$

$$7x \frac{dx}{dt} + 7y \frac{dy}{dt} = 7z \frac{dz}{dt}$$

$$\frac{17}{5} = \frac{dz}{dt}$$

5. Let $n(x) = e^{-x^2+1}$.

(a) What is the domain of $n(x)$? What are the x -intercepts? What is the y -intercept?

6

\mathbb{R}

none

$(0, e)$

(b) On which intervals is $n(x)$ increasing? On which intervals is it decreasing?

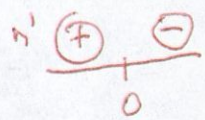
12

$$n'(x) = (-2x)e^{-x^2+1}$$

inc: $(-\infty, 0)$

critical = 0

dec: $(0, \infty)$



(c) What are the local maxima and local minima of $n(x)$?

6

$(0, e)$ is a local maximum

(d) On which intervals is $n(x)$ concave up? On which intervals is it concave down?

12

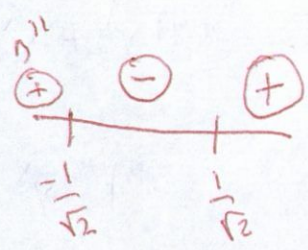
$$n''(x) = (-2x)^2 e^{-x^2+1} + -2e^{-x^2+1}$$

CU: $(-\infty, -\frac{1}{\sqrt{2}}) \cup (\frac{1}{\sqrt{2}}, \infty)$

$$= (4x^2 - 2)e^{-x^2+1}$$

CD: $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$$\text{critical} = \pm \sqrt{\frac{1}{2}}$$



(e) What are the points of inflection of $n(x)$?

6

$(-\frac{1}{\sqrt{2}}, n(-\frac{1}{\sqrt{2}}))$
" "
 $e^{1/2}$

$(\frac{1}{\sqrt{2}}, e^{1/2})$ are POI

(f) What are horizontal asymptotes and vertical asymptotes of $n(x)$?

$$\lim_{x \rightarrow \infty} e^{-x^2+1} = e^{\lim_{x \rightarrow \infty} -x^2+1} = 0$$

6

1

$y=0$ is HA

(g) Sketch the graph of $n(x)$.

4

