

Math 314 Section 2: Exam 1 Review

Disclaimer: The content for Exam 1 is everything we covered in class from chapters 1 through 6. In addition to the review problems, you should look at homework problems, quizzes, and the problems in the textbook. In general, the problems on the exam will be *similar* to the review/homework/textbook/etc. There may be things on the exam that are not included in this review sheet and vice versa.

1. You write either a 1 or a 0 in each square of a 4×4 grid. Prove that, among all rows and columns of the grid, two have the same sum.
2. How many numbers in the set $\{1, \dots, 154\}$ are relatively prime to 154? Hint 1: Use inclusion/exclusion. Hint 2: $154 = 2 \times 7 \times 11$.
3. You wake up every day and write down a random 4 letter string. How many days will it take to guarantee that you have written the same string twice?
4. Prove:
$$1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \dots + n \cdot 2^{n-1} = (n-1)2^n + 1$$
5. Prove that F_{3n} is always even. Prove that F_{5n} is divisible by 5.
6. How many anagrams of the word MATHEMATICS contain the subword MATH?
7. Prove: $\sqrt[3]{2}$ is irrational.
8. We distribute n pennies to k boys and l girls in such a way that each girl gets at least one penny (with no restriction for the boys). How many ways can this be done?
9. Does 7 have a multiplicative inverse modulo 35? How about 36?
10. Compute the greatest common divisor of 220 and 45.
11. Solve $17x \equiv 7 \pmod{35}$.
12. You randomly select an anagram of the word ULYSSES. What is the probability that your anagram ends with S?
13. Give an example of three infinite subsets of \mathbb{N} such that the intersection of all three sets is empty.