

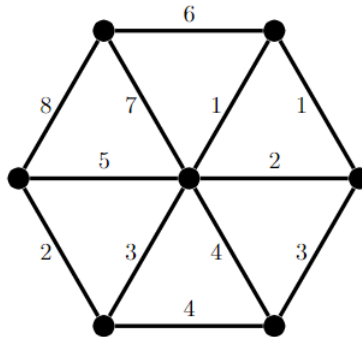
## Math 314 Section 2: Quiz 4

1. Prove that if a tree has a node of degree  $d$ , then it has at least  $d$  leaves.

Solution: Start at the degree  $d$  node and follow a path outwards along each edge touching it. Each such path will terminate when you reach a leaf, and so you have found  $d$  leaves.

2. (a) Find a spanning tree with minimum weight (and give its weight) in the following graph:

Solution: Use the greedy algorithm. Start with an edge labeled 1, then pick the other edge labeled 1, etc.



- (b) How many Hamilton cycles does the above graph have?

Solution: 6.

- (c) Find a Hamilton cycle with minimum weight.

Solution: The weight is 26. I leave it to you to check which cycle it is.

3. Prove that every tree is bipartite. Also, give an example of a bipartite graph that is not a tree.

Solution: A tree has no cycles at all, so it certainly has no odd cycles. Thus, it is bipartite.

4. True or false: If a graph has a Hamilton cycle, then it has a closed Eulerian walk. Explain.

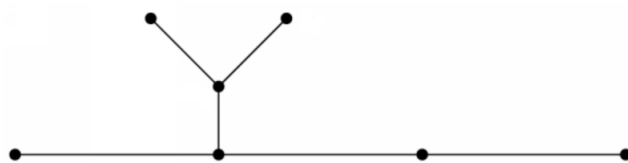
Solution: False. There are graphs that contain Hamilton cycles and have vertices of odd degree. One example is the graph drawn above.

5. True or false: If a graph has a closed Eulerian walk, then it has a Hamilton cycle. Explain.

Solution: False. For example, the graph formed from two  $K_3$  by joining them at a single vertex has a closed Eulerian walk (since all vertex degrees are even). However, it has no Hamilton cycle (it has a cut vertex).

6. Write the planar code for the following tree (use the leftmost vertex as the root):

Solution: 111010011000



7. Draw a tree that has planar code 1110011000.

Solution:

