

Math 314 Section 2: Quiz 2 solutions

1) Define Fibonacci numbers:

The fibonacci numbers F_n are the unique sequence generated by setting $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 2$.

2) Prove:

$$F_1 + F_3 + \dots + F_{2n-1} = F_{2n}.$$

Proof: We will use induction on n . For the base case, when $n = 1$ we have $F_1 = F_2 = 1$. For the inductive step, we assume that the statement is true for $n - 1$. That is, we assume:

$$F_1 + F_3 + \dots + F_{2n-3} = F_{2(n-1)} = F_{2n-2}.$$

Then,

$$\begin{aligned} F_{2n} &= F_{2n-1} + F_{2n-2} \\ &= F_{2n-1} + [F_1 + F_3 + \dots + F_{2n-3}] \\ &= F_1 + F_3 + \dots + F_{2n-3} + F_{2n-1}. \end{aligned}$$