M221 non-8:00 Sections   Test 2 v.2   Fall 2014

No books, no notes, no calculators. Write all your work on this test– nothing else will be graded. You must show your work. Your work must be legible, and your final answers must be reasonably simplified. If your answer includes arithmetic more complicated than, say, multiplying two single-digit numbers, you don't need to multiply out. (So, for instance, 4^3 does not need to be multiplied out in a final answer.)
On some problems, you are asked to use a specific method to solve the problem (for instance, "Use the definition of the derivative to find..."). On all other problems, you may use any method we've covered. **You may not use methods we have not covered. For instance, use of L'Hopital's Rule will receive no credit.**

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1. (8 points) Use Riemann sums with right endpoints to find an expression for the area under the graph as a limit. You do NOT need to evaluate the limit or do any other calculation.

$$f(x) = \sqrt{x^5+1}, \qquad 0 \le x \le 3$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{(x_i)^5+1}\,\Delta x, \qquad \Delta x = \frac{3-0}{n}$$

$$x_i = 0 + i\Delta x$$

$$x_i = \frac{i3}{n}$$

$$\boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\left(\frac{i3}{n}\right)^5+1}\cdot\frac{3}{n}}$$

2. (8 points) Find all local maxima and all local minima of $f(x) = (x-1)^5(x+4)^6$.

$$f'(x) = 5(x-1)^4(x+4)^6 + 6(x-1)^5(x+4)^5 = 0$$

$$= (x-1)^4(x+4)^5\left(5(x+4)+6(x-1)\right) = 0$$

$$= (x-1)^4(x+4)^5(11x+14) = 0$$

critical points $x = 1, -4, -14/11$

Sign of f': $+$ | -4 | $-$ | $-\frac{14}{11}$ | $+$ | 1 | $+$

so local max @ $x=-4$

local min @ $x = \frac{-14}{11}$

local max val is $f(-4)$

local min val is $f(-14/11)$

3. (21 points) Evaluate the integrals. (For indefinite integrals, give the most general indefinite integral.)

a) $\int_0^5 |2x-6|\,dx$ $= \int_0^3 -(2x-6)\,dx + \int_3^5 2x-6\,dx$

$= 6x - x^2 \Big|_0^3 + x^2 - 6x \Big|_3^5 = \boxed{13}$

b) $\int_0^2 \sqrt{4x+1}\,dx$ $\quad u = 4x+1, \ \frac{1}{4}du = dx$

$x=0, \ u=1$

$x=2, \ u=9$

$\int_1^9 u^{1/2} \cdot \frac{1}{4}\,du = \frac{1}{4}\int_1^9 u^{1/2}\,du = \frac{1}{6}u^{3/2}\Big|_1^9 = \boxed{\frac{13}{3}}$

c) $\int t(6t-3)^8\,dt$

$u = 6t-3, \quad \frac{1}{6}du = dt \qquad \frac{u+3}{6} = t$

$\int \frac{(u+3)}{6} u^8 \frac{du}{6} = \frac{1}{36}\int (u+3)u^8\,du = \frac{1}{36}\int u^9 + 3u^8\,du$

$= \frac{1}{36}\left(\frac{u^{10}}{10} + \frac{3u^9}{9}\right) + C = \boxed{\frac{1}{36}\left(\frac{(6t-3)^{10}}{10} + \frac{3(6t-3)^9}{9}\right) + C}$

4. (7 points) Find the derivative.

$\frac{d}{dx}\int_{3x+2}^{7} \sin^2(t)t^3\,dt$ $= \frac{d}{dx} - \int_7^{3x+2} \sin^2(t)\,t^3\,dt$

$= -\frac{d}{dx}\int_7^{3x+2} \sin^2(t)(t^3)\,dt = \boxed{\sin^2(3x+2)\cdot(3x+2)^3 \cdot 3}$

5. (20 points) Let $f(x) = \frac{-2x+1}{x+2}$. To save you time, I'm giving you the derivatives of f: $f'(x) = \frac{-5}{(x+2)^2}$ and $f''(x) = \frac{10}{(x+2)^3}$.

a) Find the domain of $f(x)$.

Domain $= \{x \mid x \neq -2\}$

b) Is f even? Is f odd?

Neither (won't ask this on Mid term)

c) Give all x-intercepts and all y-intercepts.

$f(0) = \frac{1}{2} \Rightarrow$ y-intercept.

$-2x-1=0$

$x = \frac{1}{2}$ x-int.

d) Give the horizontal asymptotes, or say why there are none.

Horizontal $\lim_{x \to \infty} \frac{-2x+1}{x+2} = -2.$

so H.A. at $y = -2$

e) Give the vertical asymptotes, or say why there are none.

$\lim_{x \to -2^-} \frac{-2x+1}{x+2} = -\infty$ $\lim_{x \to -2^+} \frac{-2x+1}{x+2} = +\infty$

so V.A at $x = -2$.

f) Find all intervals on which f is increasing. Find all intervals on which f is decreasing.

$f'(x) < 0$ for all x in domain so always decreasing.

g) Find all local maxima and local minima of f.

No critical points so none.

(continued on next page)

h) Find all inflection points of f.

f''

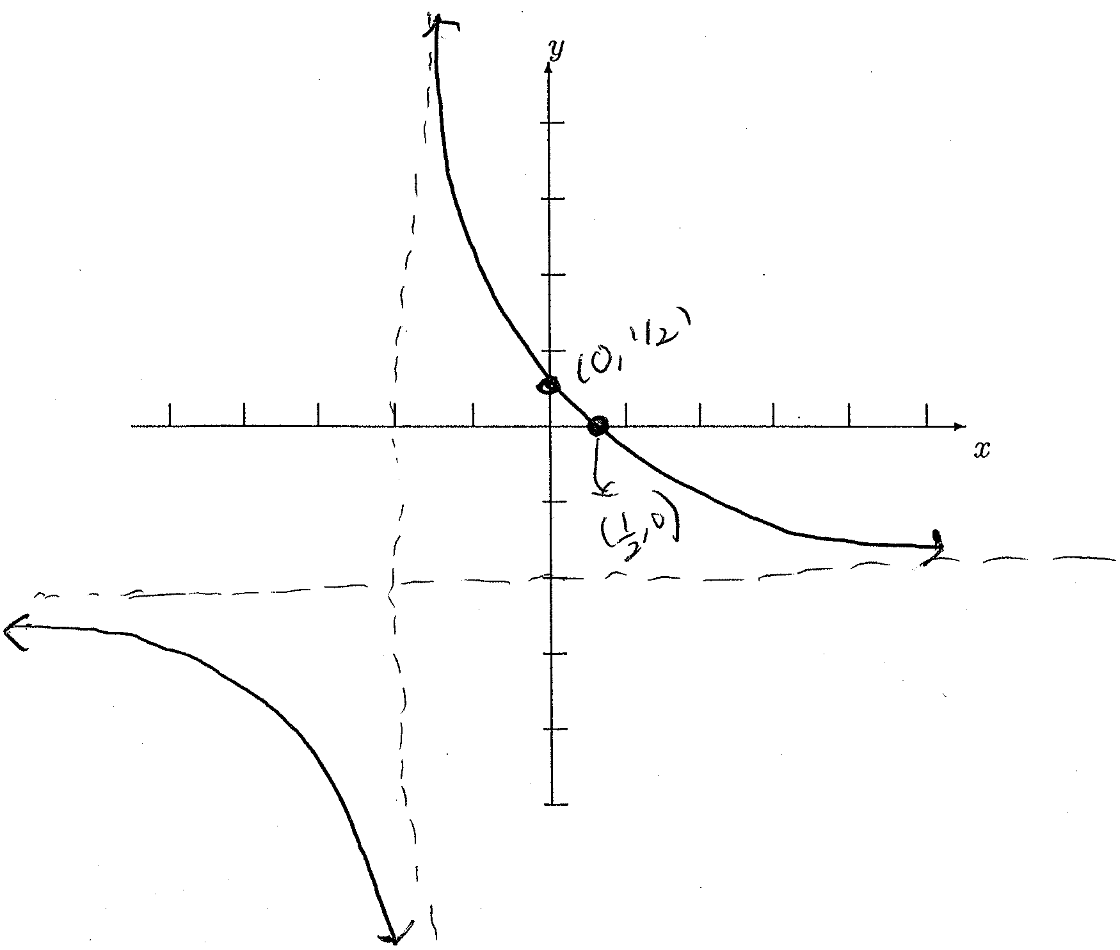
$- - - \quad | \quad + + + + +$

-2

(we'll ask you to find where concave up and down here.)

concavity changes at $x = -2$

i) Use all this information to sketch the graph of f.



6. (12 points) Let $f(x)$ and $g(x)$ be functions such that $\int_0^1 f(x)\,dx = 11$ and $\int_0^1 g(x)\,dx = 10$. Below is a list of integrals. For each part:

- If you have enough information to evaluate the integral, then do so, showing your work.
- If you don't have enough information to evaluate the integral, write "can't be done". You do not need to explain why it can't be done.

a) $\int_{-1}^{0} f(-x)\,dx$

$u = -x \qquad du = -dx$

$x = -1,\ u = 1 \qquad -du = dx$

$x = 0,\ u = 0$

$\int_1^0 f(u)(-du) = \int_0^1 f(u)\,du = 11$

b) $\int_{-1}^{1} f(x)\,dx$

Not enough info.

c) $\int_0^1 x + f(x)\,dx$

$= \int_0^1 x\,dx + \int_0^1 f(x)\,dx = \left.\frac{x^2}{2}\right|_0^1 + 11$

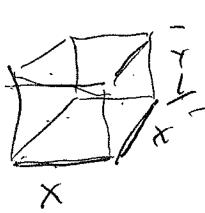
$= \boxed{\frac{1}{2} + 11}$

d) $\int_0^1 f(x)g(x)\,dx$

Not enough info.

7. (8 points) A rectangular box with a square base and lid is to be made. The volume of the box should be 20 cm^3. Material for the base and lid costs \$5 per square meter, and material for the sides costs \$3 per square meter. Find the dimensions for the box that will minimize the cost. (You must justify that your answer actually has minimal cost.)

$V = x^2 y = 20$



~~$C = 5x^2 + 3$~~

$C = 5(2x^2) + 3 \cdot 4xy$

$C = 10x^2 + 12xy$

$y = \frac{20}{x^2}, \quad C = 10x^2 + \frac{12 \cdot 20}{x} = 10x^2 + \frac{240}{x}$

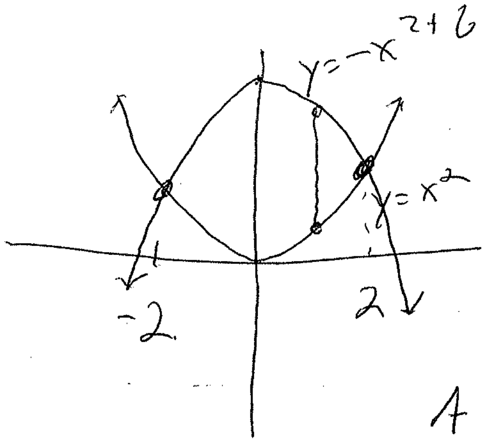
$C'(x) = 20x - \frac{240}{x^2} = \frac{20x^3 - 240}{x^2}$

$C'(x) = 0, \quad 20x^3 - 240 = 0$

$x^3 = \frac{240}{20} = 12, \quad \boxed{x = 12^{1/3}, \quad y = \frac{20}{12^{2/3}}}$

Justification: sign $C'(x)$ $\quad - - - - \underset{12^{1/3}}{|} + + + + + +$ $\quad$ so global minimum.

8. (8 points) Find the area enclosed by the curves $y = x^2$ and $y = -x^2 + 8$.



$$x^2 = -x^2 + 8$$

$$2x^2 = 8$$

$$x = \pm 2$$

$$A = \int_{-2}^{2} -x^2 + 8 - x^2 \, dx = \int_{-2}^{2} -2x^2 + 8 \, dx$$

$$= -\frac{2x^3}{3} + 8x \Big|_{-2}^{2} = \boxed{\frac{64}{3}}$$

9. (8 points) A particle is moving with acceleration $15t - 90t^2$ meters per second squared. If the initial velocity was 60 meters per second, what is the velocity after t seconds?

$$V(t) = \int a(t)\, dt + C$$

$$= \int 15t - 90t^2 \, dt$$

$$V(t) = \frac{15t^2}{2} - \frac{90}{3}t^3 + C$$

$$V(0) = 0 + C = 60$$

$$\boxed{\text{So } V(t) = \frac{15t^2}{2} - 30t^3 + 60}$$