

Find two numbers whose difference is 140 and whose product is a minimum.

Find the dimensions of a rectangle with perimeter 60 meters whose area is as large as possible.

Consider the following problem: A box with an open top is to be constructed from a square piece of cardboard, 3 feet wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

- a) Draw several diagrams to illustrate the situation, some short boxes with large bases and some tall boxes with small bases. Find the volumes of several such boxes.

- b) Draw a diagram illustrating the general situation. Let x denote the length of the side of the square being cut out. Let y denote the length of the base.

- c) Write an expression for the volume V in terms of x and y .

- d) Use the given information to write an equation that relates the variables x and y .

A rectangular storage container with an open top is to have a volume of 10 cubic meters. The length of this base is twice the width. Material for the base costs \$5 per square meter. Material for the sides costs \$3 per square meter. Find the cost of materials for the cheapest such container. (Round your answer to the nearest cent.)

A manufacturer has been selling 1000 flat-screen TVs a week at \$350 each. A market survey indicates that for each \$10 rebate offered to the buyer, the number of TVs sold will increase by 100 per week.

- Find the demand function (price p as a function of units sold x).
- How large a rebate should the company offer the buyer in order to maximize its revenue?
- If its weekly cost function is

$$C(x) = 60,000 + 120x$$

how should the manufacturer set the size of the rebate in order to maximize its profit?

A boat leaves a dock at 1 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 2 PM. How many minutes after 1 PM were the two boats closest together?

At which points on the curve

$$y = 1 + 40x^3 - 3x^5$$

does the tangent line have the largest slope?

A piece of wire 30m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle.

- a) How much wire should be used for the square in order to maximize the total area?

- b) How much wire should be used for the square in order to minimize the total area?

A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter x of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 32 ft, find the value of x so that the greatest possible amount of light is admitted.

A designer wants to introduce a new line of bookcases. He wants to make at least 100 bookcases, but not more than 2000 of them. He predicts the cost of producing x bookcases is $C(x)$. Assume that $C(x)$ is a differentiable function. Which of the following must he do to find the minimum average cost,

$$c(x) = \frac{C(x)}{x}?$$

- I find the points where $c'(x) = 0$ and evaluate $c(x)$ there
 - II compute $c''(x)$ to check which of the critical points in (I) are local maxima.
 - III check the values of c at the endpoints of its domain.
- ① I only
 - ② I and II only
 - ③ I and III only
 - ④ I, II and III

The rate (in appropriate units) at which photosynthesis takes place for a species of phytoplankton is modeled by the function

$$P = \frac{120I}{I^2 + I + 4}$$

where I is the light intensity (measured in thousands of foot-candles). For what light intensity is P a maximum?

What is the maximum vertical distance between the line

$$y = x + 6$$

and the parabola

$$y = x^2$$

for $-2 \leq x \leq 3$?

Find the area of the largest rectangle that can be inscribed in the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

An oil refinery is located 1 km north of the north bank of a straight river that is 1 km wide. A pipeline is to be constructed from the refinery to storage tanks located on the south bank of the river 6 km east of the refinery.

The cost of laying pipe is \$400,000 km over land to a point P on the north bank and \$800,000 km under the river to the tanks. To minimize the cost of the pipeline, how far downriver from the refinery should the point P be located?