

Suppose f'' is continuous on $(-\infty, \infty)$.

i) If $f'(1) = 0$ and $f''(1) = -7$, what can you say about f ?

- a) At $x = 1$, f has a local maximum.
- b) At $x = 1$, f has a local minimum.
- c) At $x = 1$, f has neither a maximum nor a minimum.
- d) More information is needed to determine if f has a maximum or minimum at $x = 1$.

ii) If $f'(3) = 0$ and $f''(3) = 0$, what can you say about f ?

- a) At $x = 3$, f has a local maximum.
- b) At $x = 3$, f has a local minimum.
- c) At $x = 3$, f has neither a maximum nor a minimum.
- d) More information is needed to determine if f has a maximum or minimum at $x = 3$.

Imagine that you are skydiving. The graph of your speed as a function of time from the time you jumped out of the plane to the time you achieved terminal velocity is

- ① increasing and concave up
- ② decreasing and concave up
- ③ increasing and concave down
- ④ decreasing and concave down

An article in the Wall Street Journal's "Heard on the Street" column (*Money and Investment* August 1, 2001) reported that investors often look at the "change in the rate of change" to help them "get into the market before any big rallies." Your stock broker alerts you that the rate of change in a stock's price is increasing. As a result you

- ① can conclude the stock's price is decreasing
- ② can conclude the stock's price is increasing
- ③ cannot determine whether the stock's price is increasing or decreasing.

True or False. If $f''(a) = 0$, then f has an inflection point at a .

For each of the following functions,

- ① Find the intervals on which the function is decreasing and the intervals on which it's increasing.
- ② Find the local minima and maxima.
- ③ Find the intervals on which it's concave up and concave down.
- ④ Find the inflection points.
- ⑤ Once you've done this, display all this information graphically on a number line.

a) $f(x) = x^4 - 8x^2 + 8$, defined on the entire real line.

b) $g(x) = 2 \cos^2(x) - 4 \sin(x)$, defined on the interval $[0, 2\pi]$

- a) Find the critical numbers of the function $f(x) = x^6(x - 2)^5$.
- b) What does the Second Derivative Test tell you about the behavior of f at these critical numbers?
- c) What does the First Derivative Test tell you that the Second Derivative test does not?

Suppose the derivative of a function f is

$$f'(x) = (x + 1)^4(x - 3)^7(x - 7)^6$$

On what intervals is f increasing?

Find the local maximum and minimum values of each function using both the First and Second Derivative Tests. Which method do you prefer?

a) $f(x) = 1 + 3x^2 - 2x^3$

b) $g(x) = \frac{x^2}{x - 1}$

c) $h(x) = \sqrt{x} - \sqrt[4]{x}$

Sketch the graph of a function that satisfies all of the given conditions:

- Vertical asymptote at $x = 0$
- $f'(x) > 0$ if $x < -2$
- $f'(x) < 0$ if $x > -2$ and $x \neq 0$
- $f''(x) < 0$ if $x < 0$
- $f''(x) > 0$ if $x > 0$

Sketch the graph of a function that satisfies all of the given conditions:

- $f'(1) = f'(-1) = 0$
- $f'(x) < 0$ if $|x| < 1$
- $f'(x) > 0$ if $1 < |x| < 2$
- $f'(x) = -1$ if $|x| > 2$
- $f''(x) < 0$ if $-2 < x < 0$
- f has an inflection point at $(0, 1)$.

Suppose

- $f(3) = 2$
- $f'(3) = \frac{1}{2}$
- $f'(x) > 0$ for all x .
- $f''(x) < 0$ for all x .

- Sketch a possible graph for f .
- How many solutions does the equation $f(x) = 0$ have? Why?
- Is it possible that $f'(2) = 1/3$? Explain.