

Sketch the graph of  $y = (x - 1)^2 + 2$  on the closed interval  $[-4, 4]$ .

- (a)** What are the local maximum and minimum values?  
points?
  
- (b)** What are the absolute maximum and minimum values?  
points?

Find the critical number of the following functions

**(a)**  $f(x) = 8x^3 - 12x^2 - 48x$

**(b)**  $g(x) = x^{\frac{3}{4}} - 9x^{\frac{1}{4}}$

**(c)**  $h(\theta) = 18 \cos(\theta) + 9 \sin^2(\theta)$

Show that 5 is a critical number of the function

$$g(x) = 2 + (x - 5)^2$$

but  $g$  does not have a local extreme value of 5.

If  $f$  has a minimum value of  $c$ , does the function  $g(x) = -f(x)$  have a maximum value of  $c$ ?

Let  $f(x)$  be a differentiable function on a closed interval with  $x = a$  being one of the endpoints of the interval. If  $f'(x) > 0$  for all  $x$ , then

- (a)  $f$  could have either an absolute maximum or minimum at  $x = a$ .
- (b)  $f$  cannot have an absolute maximum at  $x = a$ .
- (c)  $f$  must have an absolute minimum at  $x = a$ .
- (d)  $x = a$  must be a critical number for  $f$ .

If  $f$  is continuous on  $[a, b]$ , then

- (a) there must be local extreme values, but there may or may not be an absolute maximum or minimum value for the function.
- (b) there must be numbers  $m$  and  $M$  such that  $m \leq f(x) \leq M$ , for all  $x$  in  $[a, b]$ .
- (c) any absolute maximum or minimum would be at either the endpoints of the interval, or at places in the domain where  $f'(x) = 0$ .

Find the absolute extrema of:

**(a)**  $f(x) = x^3 - 3x + 1$  on the interval  $[0, 3]$ .

**(b)**  $g(x) = \frac{x^2 - 4}{x^2 + 4}$  on the interval  $[-4, 4]$ .

**(c)**  $h(t) = t\sqrt{4 - t^2}$  on the interval  $[-1, 2]$ .

**(d)**  $i(x) = x + \cot\left(\frac{x}{2}\right)$  on the interval  $\left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$ .

Find the highest and lowest points on the graph of  $f(x) = x^3 - 3x + 6$  on the following intervals:

**(i)**  $[-2, 2]$ .

**(ii)**  $[-2, 3]$ .

**(iii)**  $(-2, 3)$ .

Show that the maximum and minimum values of the function

$$f(x) = x^3 + ax^2 + bx + c$$

on the interval  $[p, q]$  occur at the endpoints if  $a^2 < 3b$ .

If  $a$  and  $b$  are positive numbers, find the maximum value of

$$f(x) = x^a(1 - x)^b$$

on the interval  $[0, 1]$ .