Steps to Solve a Related Rate

(i) What is variable?
What is constant?

(ii) Which rates are known?
Which rates need to be found?

(iii) What equation relates the variables in (ii)?

(iv) Use Implicit Differentiation on the equation in (iii) to relate the rates.

If V is the volume of a cube with edge lengths x and the cube expands as time passes, find $\frac{dV}{dt}$ in terms of $\frac{dx}{dt}$.

a) What is $\frac{dV}{dx}$ when $x = 4$ inches and is growing at a rate of 2 inches per minute?

b) What is x if the volume is shrinking at 3 cubic inches per minute and the side length is shrinking at 4 inches per minute?

c) Can a cube have a shrinking volume and a growing sides?

A spherical weather balloon is being inflated at a rate of $0.5m^3/sec$.

a) How fast is the diameter increasing at the instant the diameter is 2 meters?

b) How fast is the volume changing at that same instant?

c) How fast is the surface area changing at that same instant?

As gravel is being poured into a conical pile, its volume V changes with time. As a result, the height h and radius r also change with time. Knowing that at any moment $V = \frac{1}{3}\pi r^2 h$, the relationship between the changes with respect to time in the volume, radius and height is

1. $\dfrac{dV}{dt} = \dfrac{1}{3}\pi\left(2r\dfrac{dr}{dt}h + r^2\dfrac{dh}{dt}\right)$
2. $\dfrac{dV}{dt} = \dfrac{1}{3}\pi\left(2r\dfrac{dr}{dt}\cdot\dfrac{dh}{dt}\right)$
3. $\dfrac{dV}{dr} = \dfrac{1}{3}\pi\left(2rh + r^2\dfrac{dh}{dt}\right)$
4. $\dfrac{dV}{dh} = \dfrac{1}{3}\pi\left((r^2)(1) + 2r\dfrac{dr}{dh}h\right)$

Imagine the following magic triangle. Its base is on a horizontal surface and no matter what you do to its height, the triangle always has area 10 cm^2.

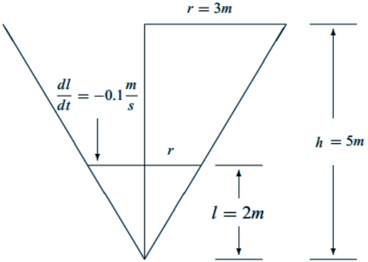
If you push down on the top of the triangle so that it becomes shorter at a rate of 3 cm/sec, how fast will the length of the base be changing when the triangle is 5 cm tall?

My neighbors have a very loud stereo. The volume knob turns half a circle (angles θ between 0° and 180°) and the volume of the music is given by the function $V(\theta) = 110\sin(\theta/2)$ decibels (dB).

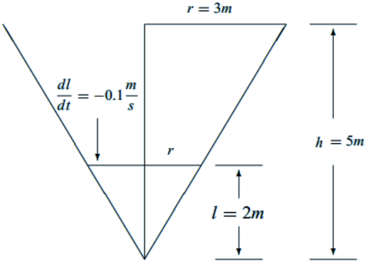
One night at 3 : 30 in the morning I notice an increase from a volume of 88 dB at a rate of 1 decibel per second! At what rate can I deduce that my neighbor is turning his volume knob?

Water is leaking out of a tank shaped like a right circular cone with height 5 m and top radius 3 m. When the water level in the cone is 2 m, the water level is decreasing at a rate of $0.1\frac{m}{s}$. How fast is the water leaking out of the cone?

Water is leaking out of a tank shaped like a right circular cone with height 5 m and top radius 3 m. When the water level in the cone is 2 m, the water level is decreasing at a rate of $0.1\frac{m}{s}$. How fast is the water leaking out of the cone?

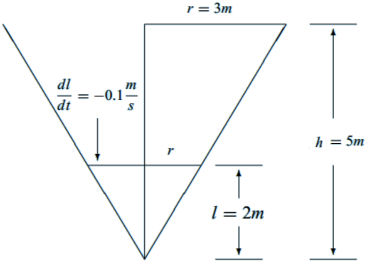


Water is leaking out of a tank shaped like a right circular cone with height 5 m and top radius 3 m. When the water level in the cone is 2 m, the water level is decreasing at a rate of $0.1\frac{m}{s}$. How fast is the water leaking out of the cone?



The volume of the water in the cone is $V = \frac{1}{3}\pi r^2 h$ and using the figure above and similar triangles $\frac{r}{h} = \frac{3}{5}$

Water is leaking out of a tank shaped like a right circular cone with height 5 m and top radius 3 m. When the water level in the cone is 2 m, the water level is decreasing at a rate of $0.1\frac{m}{s}$. How fast is the water leaking out of the cone?



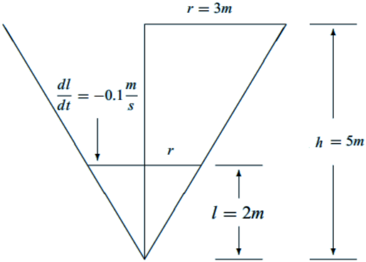
The volume of the water in the cone is $V = \frac{1}{3}\pi r^2 h$ and using the figure above and similar triangles $\frac{r}{h} = \frac{3}{5}$, which means

$$r = \frac{3}{5}h = \frac{3}{5}2 = \frac{6}{5}.$$

This means that

$$V = \frac{1}{3}\left(\frac{6}{5}\right)^2 \pi h = \frac{12\pi}{25}h.$$

Water is leaking out of a tank shaped like a right circular cone with height 5 m and top radius 3 m. When the water level in the cone is 2 m, the water level is decreasing at a rate of $0.1\frac{m}{s}$. How fast is the water leaking out of the cone?



The volume of the water in the cone is $V = \frac{1}{3}\pi r^2 h$ and using the figure above and similar triangles $\frac{r}{h} = \frac{3}{5}$, which means

$$r = \frac{3}{5}h = \frac{3}{5}2 = \frac{6}{5}.$$

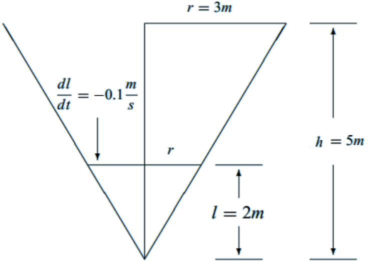
This means that

$$V = \frac{1}{3}\left(\frac{6}{5}\right)^2 \pi h = \frac{12\pi}{25}h.$$

Taking the derivative with respect to time

$$\frac{dV}{dt} = \frac{12\pi}{25}\frac{dh}{dt} = \frac{12\pi}{25}\frac{1}{10} = \frac{6\pi}{125}\frac{m^3}{s}.$$

Water is leaking out of a tank shaped like a right circular cone with height 5 m and top radius 3 m. When the water level in the cone is 2 m, the water level is decreasing at a rate of $0.1\frac{m}{s}$. How fast is the water leaking out of the cone?



The volume of the water in the cone is $V = \frac{1}{3}\pi r^2 h$ and using the figure above and similar triangles $\frac{r}{h} = \frac{3}{5}$, which means

$$r = \frac{3}{5}h \quad \cancel{= \frac{3}{5}2 = \frac{6}{5}}.$$

This means that

$$V = \frac{1}{3}\left(\frac{3}{5}h\right)^2 \pi h = \frac{3\pi}{25}h^3.$$

Taking the derivative with respect to time

$$\frac{dV}{dt} = \frac{3\pi}{25}3h^2\frac{dh}{dt} = \frac{3\pi}{25}3(2)^2\frac{-1}{10} = \frac{-36\pi}{250}\frac{m^3}{s}.$$

a) A streetlight hangs 5 meters above the ground. Regina, who is 1.5 meters tall, walks away from the point under the light at a rate of 2 meters per second. How fast is her shadow lengthening when she is 7 meters away from the point under the light?
(Hint: Use similar triangles.)

b) Suppose Regina has the ability to magically shrink herself. At what rate must she do this to keep her shadow a constant length of 3 meters? Write this as a function of only her distance from the point under the light.

A revolving beacon from a light house shines on the straight shore, and the closest point on the shore is a pier one half mile from the lighthouse. Let θ denote the angle between the lighthouse, pier, and point on the shore where the light shines.

1. Write the distance from the pier to the point of light as a function of θ.
2. What is the rate of change of the distance from the pier to the point of light with respect to θ.
3. Suppose θ is a function of time t. Give an expression for the rate of change of distance with respect to time t.
4. Suppose that the light makes 1 revolution per minute. How fast is the light traveling along the straight beach at the instant it pases over a shorepoint 1 mile away from the shorepoint nearest the searchlight?

Given that a spherical raindrop evaporates at a rate proportional to its surface area, how fast does the radius shrink?

The minute hand on a watch is 8 mm long and the hour hand is 4 mm long. How fast is the distance between the tips of the hands changing at one o'clock?

a) Find the shortest line segment with endpoints on the x and y axes going through the point $(1, 8)$.

b) What is the area of the triangle formed by the shortest line segment?

c) What is the rate of change of area with respect to the x-coordinate of the point on the x-axis?

d) For which x is the area increasing?

The speed limit on a stretch of highway is 55 mph. Highway patrol officer, Sgt. Miguel, stations himself at a point, out of view of the motorists, 50 feet off the highway. Miguel is equipped with a radar gun which measures the speed at which a car approaches **his position**.

He takes a reading of suspected speeders by pointing his radar gun at a point on the highway 120 feet from the point on the highway closest to him. The radar gun picks up a reading of 48 feet/sec for a green Chevy driven by Alyssa. How fast is she traveling? Is Alyssa speeding?

At a certain moment, ship A is 6 miles south and 8 miles west of ship B . Ship A at that moment is steaming east at 12 mph, while ship B is steaming north at 15 mph.

Are the ships approaching each other or separating from each other? At what rate?

Particle A moves along the positive horizontal axis, and particle B along the graph of

$$f(x) = -\sqrt{3}x, \; x \leq 0.$$

At a certain time, A is at the point $(5, 0)$ and moving with speed 3 units/sec; and B is at a distance of 3 units from the origin moving with speed 4 units/sec. At what rate is the distance between A and B changing?