

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1 \quad \lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x} = 0$$

Evaluate the limits:

$$\lim_{x \rightarrow 0} \frac{\sin(6x)}{\sin(9x)}$$

$$\lim_{x \rightarrow 0} \frac{\sin(x^7)}{x}$$

$$\lim_{\theta \rightarrow 0} \frac{\cos(7\theta) - 1}{\sin(9\theta)}$$

$$\lim_{x \rightarrow 2} \frac{\sin(x - 2)}{x^2 + 6x - 16}$$

$$\lim_{t \rightarrow 0} \frac{\tan(16t)}{\sin(4t)}$$

$$\lim_{x \rightarrow 2} \frac{3 - 3 \tan(x)}{\sin(x) - \cos(x)}$$

Use both the derivatives of  $\sin(x)$  and  $\cos(x)$  and the quotient rule to show:

$$\frac{d}{dx} (\tan(x)) = \sec^2(x)$$

and

$$\frac{d}{dx} (\sec(x)) = \sec(x) \tan(x)$$

Find  $f'(x)$ :

$$f(x) = 5x^2 + 7 \sin(x)$$

Find  $F'(x)$ :

$$F(x) = \frac{3 - \sec(x)}{\tan(x)}$$

Find  $g'(\theta)$ :

$$g(\theta) = \sec(\theta) \tan(\theta)$$

Evaluate:

$$\frac{d^2}{d\theta^2} (\theta \sin(\theta))$$

Find the equation of the tangent line to the curve  $y = 14x \sin(x)$  at  $x = \pi/2$ .

Find

$$\frac{d^{103}}{dx^{103}} (\sin(x)) \quad \text{and} \quad \frac{d^{201}}{dx^{201}} (\cos(x))$$

*Step into your teacher's shoes. What is wrong (if anything) with the following calculations? Explain any errors and correct for them.*

Find all values of  $x$  in the interval  $[0, 4\pi)$  that satisfy the equation

$$\sin(2x) = \cos(x).$$

**Solution:** Since  $\sin(2x) = 2 \sin(x) \cos(x)$ ,

$$\sin(2x) = \cos(x) \quad \Rightarrow \quad 2 \sin(x) \cancel{\cos(x)} = \cancel{\cos(x)}$$

$$\Rightarrow \quad 2 \sin(x) = 1 \quad \Rightarrow \quad \sin(x) = \frac{1}{2}$$

Therefore,  $x = \frac{\pi}{3}$  or  $x = \frac{5\pi}{3}$ .

What does  $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$  mean? Explain why three of the options are false and one is true.

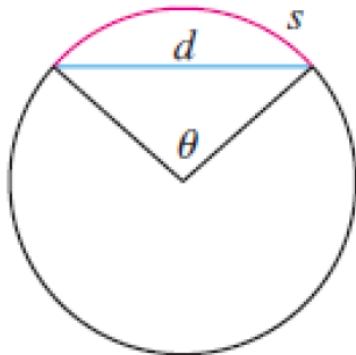
a)  $\frac{0}{0} = 1$ .

b) The tangent to the graph of  $y = \sin(x)$  at  $(0, 0)$  is the line  $y = x$ .

c) You can cancel the  $x$ 's.

d)  $\sin(x) = x$  for  $x$  near 0.

The figure shows a circular arc of length  $s$  and a chord of length  $d$ , both subtended by a central angle  $\theta$ . Find  $\lim_{\theta \rightarrow 0^+} \frac{s}{d}$ .



It may be helpful to review the formulas associated to arcs and isosceles triangles. Further, why would the limit  $\lim_{\theta \rightarrow 0} \frac{s}{d}$  not exist?