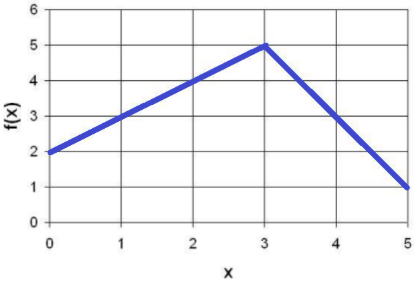
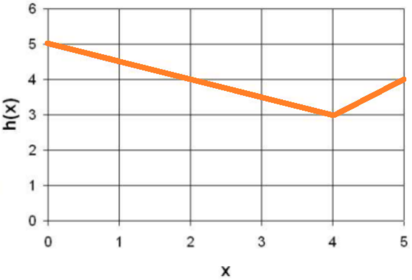
The functions $f(x)$ and $h(x)$ are graphed below:





1. Graph the function $g(x) = (2f - h)(x)$.
2. Label the slopes along the line segments of f, g, and h.
3. Plot f', g', and h'. Do these graphs agree with the differentiation rules? How are these derivatives related?

Assume the functions f and g are such that:

$$f(5) = 1 \qquad f'(5) = 9$$

$$g(5) = -4 \qquad g'(5) = 5$$

Evaluate the following expressions:

(a) $(f + g)'(5)$

(b) $(fg)'(5)$

(c) $(f/g)'(5)$

(d) $(g/f)'(5)$

(e) $\frac{d}{dx}\left(\frac{g(x)}{x}\right)$ at $x = 5$.

Easier problems:

$$f(x) = x^4 - 2x^2 + 6$$

$$g(x) = 7x + 4x^{-1/8}$$

$$h(x) = \frac{x^4}{5 - x^3}$$

Harder problems:

$$F(x) = \sqrt{x}(x - 4)$$

$$G(x) = \frac{8x^2 + 2x + 4}{\sqrt{x}}$$

$$H(u) = \sqrt{6}u + \sqrt{5u}$$

The Constant Multiple Rule tells us

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$$

and the Product Rule says

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x)) + f(x)\frac{d}{dx}(c).$$

Why do these agree?

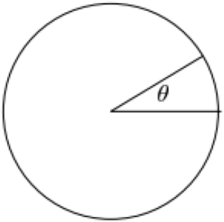
Find the first and second derivatives.

(a) $f(x) = 2x^4 - 2x^3 + 4x$

(b) $g(r) = \sqrt{r} + \sqrt[3]{r}$

(c) $h(x) = \dfrac{x^2}{1 + 8x}$

Suppose you cut a slice of pizza from a circular pizza of radius r,



As you change the size of the angle θ, you change the area of the slice, $A = \frac{1}{2}r^2\theta$. Then A' is

(a) $r\theta$ (b) $\frac{1}{2}r^2$ (c) r (d) Unknowable

Find an equation of the tangent line to the curve

$$y = \frac{5x}{x+3}$$

at the point $(2, 2)$.

Find the points on the curve

$$y = 2x^3 + 3x^2 - 12x + 3$$

where the tangent line is horizontal.

The equation of motion of a particle is

$$s = t^3 - 27t$$

where s is in meters and t is in seconds. (Assume $t \geq 0$.)

(a) Find the velocity and acceleration as functions of t.

(b) Find the acceleration when the velocity is zero.

Use the product rule to show

$$\frac{d}{dx}(fgh)(x) = (fgh' + fhg' + ghf')(x)$$

Can you generalize this argument? ie What is the derivative of four, five, six, ... functions multiplied together?

Find the derivative of

$$y = (3x + 1)(2x - 1)(x - 2) \qquad y = (3x - 2)^2(2x + 3)$$

Consider the function $f(x) = |x^2 - 25|$.

1. Sketch the graph of f.
2. Find a formula for f'.
3. For what values of x is the function not differentiable?

What is the derivative of the function $f \cdot f$? What is the derivative of the function $f \cdot f \cdot f$? Can you generalize this for any positive integer n? What is the derivative of $(x^2 + 1)^6$?

The functions

$$y = x^2 + ax + b \qquad y = cx - x^2$$

share a tangent line at the point $(1, 0)$. Find a, b, and c.