

If  $f(x) = \frac{x^2 - 4}{x - 2}$  and  $g(x) = x + 2$ , then we can say the functions  $f$  and  $g$  are equal.

$$\lim_{x \rightarrow 2} f(x) = 4 \quad \lim_{x \rightarrow 2} g(x) = -2 \quad \lim_{x \rightarrow 2} h(x) = 0$$

Find the limits, if they exist:

$$\text{(a)} \quad \lim_{x \rightarrow 2} [f(x) + 5g(x)]$$

$$\text{(d)} \quad \lim_{x \rightarrow 2} 4f(x)g(x)$$

$$\text{(b)} \quad \lim_{x \rightarrow 2} [g(x)]^3$$

$$\text{(e)} \quad \lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$$

$$\text{(c)} \quad \lim_{x \rightarrow 2} \frac{1}{f(x)}$$

$$\text{(f)} \quad \lim_{x \rightarrow 2} \frac{g(x)h(x)}{f(x)}$$

$$\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x - 1}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{9 + h} - 3}{h}$$

$$\lim_{x \rightarrow -1} \frac{x^2 - 4x}{x^2 - 3x - 4}$$

$$\lim_{x \rightarrow -4} \frac{1/4 + 1/x}{4 + x}$$

$$\lim_{h \rightarrow 0} \frac{(-4 + h)^2 - 16}{h}$$

$$\lim_{x \rightarrow 0} \frac{9}{t} - \frac{9}{t^2 + t}$$

## True or False.

Consider a function  $f(x)$  with the property that  $\lim_{x \rightarrow a} f(x) = 0$ . Now consider another function  $g(x)$  also defined near  $a$ . Then

$$\lim_{x \rightarrow a} [f(x)g(x)] = 0$$

## True or False.

If  $\lim_{x \rightarrow a} f(x) = \infty$  and  $\lim_{x \rightarrow a} g(x) = \infty$ , then

$$\lim_{x \rightarrow a} [f(x) - g(x)] = 0$$

Find the following limits.

$$\lim_{x \rightarrow 3} 8x + |x - 3|$$

$$\lim_{x \rightarrow -3} \frac{4x + 12}{|x + 3|}$$

If

$$2x - 2 \leq f(x) \leq x^2 - 2x + 2$$

for  $x \geq 0$ , find  $\lim_{x \rightarrow 2} f(x)$ .

Consider the function

$$f(x) = \begin{cases} x^2 & x \text{ is rational, } x \neq 0 \\ -x^2 & x \text{ is irrational} \\ \text{undefined} & x = 0 \end{cases}$$

Then

- 1 there is no  $a$  for which  $\lim_{x \rightarrow a} f(x)$  exists
- 2 there may be some  $a$  for which  $\lim_{x \rightarrow a} f(x)$  exists, but it is impossible to say without more information
- 3  $\lim_{x \rightarrow a} f(x)$  exists only when  $a = 0$
- 4  $\lim_{x \rightarrow a} f(x)$  exists for infinitely many  $a$