§1.4 and §2.1 Derivatives and Rates of Change

The cost (in dollars) of producing x units of a certain commodity is $C(x) = 5000 + 6x + 0.05x^2$.

Find the average rate of change of C with respect to x when the production level is changed from $x = 100$ to the given value. (Round your answers to the nearest cent.)

a) $x = 103$

b) $x = 101$

Each limit below represents the derivative of some function f at some number a, find them.

a) $\displaystyle \lim_{h\to 0} \frac{(16+h)^{1/4}-2}{h}$

b) $\displaystyle \lim_{x\to \pi/4} \frac{\tan(x)-1}{x-\pi/4}$

c) $\displaystyle \lim_{t\to 1} \frac{t^5+t-2}{t-1}$

The number of gallons of water in a tank t minutes after the tank has started to drain is $Q(t) = 200(30 - t)^2$.

a) What is the average rate at which the water flows out
   - during the first ten minutes?
   - during the five minutes from $t = 5$ to $t = 10$?
   - during the two minutes from $t = 8$ to $t = 10$?
   - during the minute from $t = 9$ to $t = 10$?

b) Estimate how fast the water is running out of the tank at the end of ten minutes.

c) Draw a graph of the function Q for $0 \leq t \leq 20$. Draw the secant lines for the four time intervals used in Part a). What are their slopes?

The cost (in dollars) of producing x units of a certain commodity is $C(x) = x^2 - 2x + 10$.

a) Find the average rate of change of C with respect to x when the production level is changed from $x = 5$ to $x = 7$ and when changed from $x = 5$ to $x = 6$.

b) Find the instantaneous rate of change of C with respect to x when $x = 5$.