NOTE: In the following pages are 5 sample final exams given to past 323 students by the Calc 3 coordinator Dr. Bill Kazmierczak. The purpose of these exams is to help review material. Some problems on these exams were not covered in class so are redacted. IMPORTANT: Please do not only study these problems! There are a great variety of problems we did this semester and your final exam may have different types of problems!

Calculus 3 Final Sample Exam 1

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 3. Find all critical points of the function $f(x,y) = 4x - 3x^3 - 2xy^2$. For each critical point determine if it is a local maximum, local minimum or a saddle point.

Problem 4. Find the volume of the solid E bounded by $y = x^2$, $x = y^2$, $z = x+y+5$, and $z = 0$.

Problem 5. Find the integral of the function $f(x,y) = y^2$ on the region bounded by $y^2 = x+4$, $x = 0$, and $y \geq 0$. Simplify your answer as much as possible.

Problem 7. Evaluate the line integral $\oint_C e^{2x+y}\,dx + e^{-y}\,dy$ along the **negatively** oriented closed curve C, where C is the boundary of the triangle with the vertices $(0,0)$, $(0,1)$, and $(1,0)$.

Problem 9. Evaluate the line integral $\oint_C \mathbf{F}\cdot d\mathbf{r}$ for the vector field $\mathbf{F}(x,y,z) = -y\,\mathbf{i} + x\,\mathbf{j} - z\,\mathbf{k}$, where the closed curve C is the boundary of the triangle with vertices $(0,0,5)$, $(2,0,1)$, and $(0,3,2)$ traced in this order.

Problem 10. Evaluate the flux of $\mathbf{F}(x,y,z) = z^2y\,\mathbf{i} + x^2y\,\mathbf{j} + (x+y)\,\mathbf{k}$ over S, where S is the closed surface consisting of the coordinate planes and the part of the sphere $x^2+y^2+z^2 = 4$ in the first octant $x \geq 0$, $y \geq 0$, $z \geq 0$, with the normal pointing outward.

Calculus 3 Final Examination Sample 1 - ANSWERS

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 3. $(0, \sqrt{2})$ and $(0, -\sqrt{2})$ - saddles;
$(\frac{2}{3}, 0)$ - local maximum
$(-\frac{2}{3}, 0)$ - local minimum

Problem 4. $\frac{59}{30}$

Problem 5. $\frac{64}{15}$

Problem 7. $\frac{e^2}{2} - e + \frac{1}{2}$

Problem 9. 6.

Problem 10. $\frac{16\pi}{15}$

Calcululus 3 Final Sample Exam 2

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. Function f is given by the formula $f(x, y) = 2x^2 + 3e^{xy}$.

a) Find the directional derivative of f at the point $P = (1, 0)$ in the direction of the vector $\mathbf{u} =< -1, 2 >$.

b) Find the maximal rate of change of $f(x, y)$ at P and the direction in which it occurs.

Problem 2. The curve is given parametrically by
$\mathbf{r}(t) =< t^3 + \frac{1}{2}t^2, 2t - 1, t^2 + t\sqrt{5} >$.

Set up the integral representing the length of the curve from the point $(0, -1, 0)$ to the point $(10, 3, 4 + 2\sqrt{5})$.
DO NOT EVALUATE THE INTEGRAL.

Problem 3. Find an equation of the plane tangent to the surface
$x^2 + y^2z^2 = 8$ at the point $P = (2, 2, 1)$.

Problem 4. Find all critical points of the function
$f(x, y) = x^2 + 4xy - 10x + y^2 - 8y + 1$. For each critical point determine if it is a local maximum, a local minimum or a saddle point.

Work $= \int \mathbf{F} \cdot d\mathbf{r}$

Problem 5. Find the work done by the force $\mathbf{F}(x, y) = 3y\mathbf{i} + x\mathbf{j}$ in moving a particle along the boundary of the trapeziod with the vertices $(0, 0)$, $(1, 1)$, $(2, 1)$ and $(3, 0)$ in the clockwise direction.

Problem 6. Find $\iiint_E \rho(x,y,z)\,dV$, where E is the solid bounded by the surfaces $y^2 + z^2 = 1$, $x = 0$ and $x = y^2 + z^2 - 4$, and $\rho(x,y,z)$ is given by the formula $\rho(x, y, z) = y^2 + z^2$.

Problem 7. a) Determine whether the vector field
$\mathbf{F}(x, y, z) = (2y + 4z)\mathbf{i} + (2x + 3z)\mathbf{j} + (4x + 3y)\mathbf{k}$, is conservative or not.

b) Evaluate $\int_C (2y + 4z)dx + (2x + 3z)dy + (4x + 3y)dz$, where C is the curve given by $\mathbf{r}(t) =< t^3, 2\sin\left(\frac{\pi t}{2}\right), 3\cos\left(\frac{\pi t}{2}\right) >$ for $0 \le t \le 1$.

Problem 8. Find the maximum and minimum values of the function $F(x, y, z) = x - y$ on the $x^2 + y^2 + xy + z^2 = 1$

Problem 9. Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, if $\mathbf{F}(x, y, z) = y\mathbf{i} + 2x\mathbf{j} + yz\mathbf{k}$, and C is the curve of intersection of the part of the paraboloid $z = 1 - x^2 - y^2$ in the first octant $(x \ge 0,\ y \ge 0,\ z \ge 0)$ with the coordinate planes $x = 0$, $y = 0$ and $z = 0$, oriented counterclockwise when viewed from above.

Problem 10. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, if $\mathbf{F}(x, y, z) = (yz)\mathbf{i} + (x^2y)\mathbf{j} + (4zx^2)\mathbf{k}$ and S is the surface of the solid bounded by the upper hemisphere
$x^2 + y^2 + z^2 = 1$, $z \ge 0$, and the plane $z = 0$ with the normal pointing outward.

Calculus 3 Final Examination Sample 2 - ANSWERS

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. a) $\frac{2}{\sqrt{5}}$
b)$< \frac{4}{5}, \frac{3}{5} >$ (or $< 4, 3 >$)

Problem 2.
$$\int_0^2 \sqrt{(3t^2+t)^2+4+(2t+\sqrt{5})^2}\, dt$$

Problem 3. $(x-2)+(y-2)+2(z-1)=0$

Problem 4. $(1,2)$ - saddle point

Problem 5. 4

Problem 6. $\frac{5\pi}{3}$

Problem 7. a) conservative;
b) 4

Problem 8. Absolute minimum is -2 at $(-1,1,0)$;
Absolute maximum is 2 at $(1,-1,0)$

Problem 9. $\frac{\pi}{4}+\frac{4}{15}$

Problem 10. $\frac{2\pi}{3}$

Calculus 3 Final Sample Exam 3

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 3. Determine all local maxima, local minima and saddle points of $f(x, y) = 3y - y^3 - 3x^2y$.

Problem 4. A rectangular box without a lid is to be made from 48 ft^2 of cardboard. Find the maximum volume of the box.

Problem 5. Find $\iint_R y^2 dA$ and $\iint_R y\, dA$, where R is the region bounded by $y^2 = x + 4$, $x = 0$, and $y \geq 0$.
Divide your answers for the integrals.
[Your final answer equals the quotient $\dfrac{\iint_R y^2 dA}{\iint_R y\, dA}$.]

Problem 6. Find the volume of the solid that lies within the cylinder $x^2 + y^2 = 4$, above the $(x, y)-$plane, and below the cone $z^2 = 4x^2 + 4y^2$.

Problem 7. Let $\mathbf{F}$ be the two-dimensional vector field given by $\mathbf{F}(x, y) =< ye^{xy} - 1, xe^{xy} + 2y >$.

a) Determine if $\mathbf{F}$ is a conservative vector field, and if so, find a potential function.

Note: $\int_C \mathbf{F}\cdot\mathbf{T}ds = \int_C \mathbf{F}\cdot d\mathbf{r}$

b) Find the value of the line integral $\int_C \mathbf{F}\cdot\mathbf{T}ds$, where C is the line segment from $(0, 3)$ to $(5, 0)$.

Problem 8. Find the value of $\oint_C -5x^2\, dx + 7xy\, dy$, where C is the closed curve consisting of the edges of the triangle with vertices $(0, 0)$, $(3, 1)$, and $(0, 3)$, oriented counterclockwise.

Problem 9. Find the total flux $\iint_S \mathbf{F}\cdot\mathbf{n}\, dS$ of the vector field $\mathbf{F}(x, y, z) =< x^2, yz^2, -2xz >$ across the surface S given by $x^2 + y^2 + z^2 = 2$ with outward orientation.

Problem 10. Evaluate $\oint_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = e^x\mathbf{i} + (x^2 + y^2)\mathbf{j} + z\mathbf{k}$, and C is the boundary of the part of the plane $2x + y + 2z = 2$ in the first octant oriented counterclockwise when viewed from above.

Calculus 3 Final Examination Sample 3 - ANSWERS

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 3. $(1,0)$ and $(-1,0)$ - saddles;
$(0,1)$ - local maximum
$(0,-1)$ - local minimum

Problem 4. $32\ ft^2$

Problem 5. $\frac{16}{15}$

Problem 6. $\frac{32\pi}{3}$

Problem 7. a) conservative; $f(x,y) = e^{xy} + y^2 - x$
b) -14

Problem 8. 42

Problem 9. $\frac{16\pi\sqrt{2}}{15}$

Problem 10. $\frac{2}{3}$

Calculus 3 Final Sample Exam 4

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. The points $A(2,1,3), B(0,5,5), C(3,6,9)$ and $D(5,2,7)$ are the vertices of a parallelogram P. Calculate the area of P.

Problem 2. Determine whether the lines
$L_1 : x = t+2, y = 3t+1, z = t+3,\ L_2 : x = -2s+3, y = -2, z = 4s-2,$
intersect. If they intersect, find the point(s) of intersection. If they do not intersect, are they parallel?

Problem 3. Let S be the level surface defined by $x^2 - y^2 + z^2 = 1$.

(a) Find the equations of the tangent plane and normal line to S at the point $(1,1,-1)$.

(b) Find all the points of intersection of the line found in (a) and the surface S (of course, $(1,1,-1)$ is one of them).

Problem 4. The position of a particle at time t is given by the function $\mathbf{r}(t) = \left\langle \frac{1}{3}t^3, 2t\sin t, 2t\cos t \right\rangle, t \geq 0$. Find the speed $v(t)$ of the particle and the distance traveled (arc length) between the times $t = 0$ and $t = 3$.

Problem 5. Let $z = xy, x = uv, y = vw$. Use the Chain Rule to find $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ and $\frac{\partial z}{\partial w}$. Give your answers in terms of the variables u, v and w alone.

Problem 6. Let $f(x,y) = (x-3)^2 + (y+3)^2$.

(a) Find the minimum and maximum values of f under the constraint $x^2 + y^2 = 8$.

(b) Find the minimum and maximum values of f in the disk with center $(0,0)$ and radius $\sqrt{8}$.

Problem 7. Evaluate $\int\int_D 4y^2 dA$ where D is the intersection of the unit disk with the first quadrant: $D = \{(x,y) | x \geq 0, y \geq 0, x^2 + y^2 \leq 1\}$.

Problem 8. Determine whether the field $\mathbf{F}$ is conservative. If it is, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the plane curve with equation $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$, $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

(a) $\mathbf{F}(x,y) = x^2 \cos y\, \mathbf{i} + 2x \sin y\, \mathbf{j}$.

(b) $\mathbf{F}(x,y) = (\sin x + \cos y)\, \mathbf{i} + (2 - x\sin y)\, \mathbf{j}$.

Problem 9. Let D be a plane region whose boundary C is a simple smooth closed curve. Assume that the area of D is 3 and that C is given the clockwise orientation. Evaluate $\int_C (xy^2 + 2y)dx + x^2 y dy$. (You are not allowed to make any specific choice for D.)

Problem 10. Find the flux $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ of the constant vector field $\mathbf{F}(x,y,z) = \langle 0,0,1 \rangle$ across the surface S with vector equation
$\mathbf{r}(u,v) = \langle u^2 - v, u + v^2, uv \rangle, 0 \leq u \leq 1, 0 \leq v \leq 1$.

Calculus 3 Final Examination Sample 4 - ANSWERS
courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1.
The area is $14\sqrt{3}$.

Problem 2. The lines intersect at $(1, -2, 2)$.

Problem 3. a) Plane: $x - y - z = 1$;
line: $x = 1 + t,\ y = 1 - t,\ z = -1 - t$
b) $(1, 1, -1)$ and $(-1, 3, -3)$.

Problem 4. $v(t) = t^2 + 2$

$d = 15$

Problem 5. $\frac{\partial z}{\partial u} = v^2 w, \quad \frac{\partial z}{\partial v} = 2uvw, \quad \frac{\partial z}{\partial w} = uv^2$

Problem 6. Answer for both parts a) and b):
Absolute maximum is 50 at $(-2, 2)$;
Absolute minimum is 2 at $(2, -2)$

Problem 7. $\frac{\pi}{8} - \frac{1}{4}$

Problem 8. a) Not conservative;
b) Conservative; 4

Problem 9. 6

Problem 10. 2

Calculus 3 Final Sample Exam 5

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. Find the points on the cone $z^2 = x^2 + y^2$ closest to the point $(1, \sqrt{3}, 4)$.

Problem 2. Express the volume of the tetrahedron with vertices at points $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, $(0,0,3)$ as a triple integral. Write out the limits of integration explicitly, but do not evaluate.

Problem 3. Write the equation of the plane tangent to the cone $z^2 = x^2 + y^2$ at the point $(1, 0, 1)$.

Problem 4. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $F(x,y,z) = e^x\mathbf{i} + (x^2+y)\mathbf{j} + z\mathbf{k}$, and C is the boundary of the part of the plane $2x + y + 2z = 2$ in the first quadrant oriented counterclockwise as viewed from above.

Problem 5. Consider the helix $r(t) =< \cos t, \sin t, t >$, $0 \le t \le \pi$.
Find the length of this helix.

Problem 6. Let D be the region $D = \{(x,y,z) : x^2 + y^2 + z^2 \le 2,\ z \ge x^2 + y^2\}$ and S be the surface of D with outward pointing normal.

a) Calculate the volume of D.

b) Let $\mathbf{F}(x,y,z) =< yz^2, 3x + z\cos x, x^2y^3 >$. Use the divergence theorem and the result in part a) to find $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

Problem 7. Let S be the lateral surface of the cone $z^2 = x^2 + y^2$, between $z = 0$ and $z = 1$ with outward orientation, and let $\mathbf{F}(x,y,z) =< -y, x, 0 >$. Denote by I the value of the integral $\iint_S curl(\mathbf{F}) \cdot d\mathbf{S}$.

a) Compute $curl(\mathbf{F})$ and $div(\mathbf{F})$.

b) Find I by direct evaluation.

c) Find I by using Stokes' theorem.

d) Find I by use of the divergence theorem. (Hint: Cover the top of the cup by a flat circular disk.)

Problem 8. Find two non-zero vectors $\mathbf{v}$ and $\mathbf{w}$, such that $\mathbf{v} \cdot \mathbf{w} = 0$ and both $\mathbf{v}$ and $\mathbf{w}$ are perpendicular to the line $x = y = z$.

Problem 9. Find the limit, if it exists:

$$\lim_{(x,y)\to(0,0)} \frac{2009x^2 - y^2}{1 - \sqrt{1 + 2009x^2 - y^2}}$$

Problem 10. Let $\mathbf{F}(x,y,z) =< xy - 3, -y^2, yz - z >$. Evaluate in the simplest way possible the surface integral $\int_S\int \mathbf{F} \cdot \mathbf{n} dS$, where S is the boundary of the unit cube $\{(x,y,z) : 0 \le x, y, z \le 1\}$, and $\mathbf{n}$ denotes the inward unit normal to S.

Calculus 3 Final Examination Sample 5 - ANSWERS

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. $(\frac{3}{2}, \frac{3\sqrt{3}}{2}, 3)$

Problem 2. There are 6 possible correct answers. One of the possible answers is:

$$\int_0^1 \int_0^{(2-2x)} \int_0^{(1-\frac{y}{2}-\frac{z}{3})} dz\, dy\, dx$$

Problem 3.

$$x - z = 0.$$

Problem 4. $\frac{2}{3}$

Problem 5. $\sqrt{2}\pi;$

Problem 6. a) $\frac{4\pi\sqrt{2}}{3} - \frac{7\pi}{6}$; b) 0.

Problem 7. a) $< 0, 0, 2 >$ and 0; The answer for b),c), d): $\frac{4\pi}{3}$

Problem 8. There are many possible correct answers here. A possible choice: $\mathbf{v} =< 1, -1, 0 >$ and $\mathbf{w} =< 1, 1, -2 >$

Problem 9. -2

Problem 10. 0