

Math 227 Sample Final Examination 1 Answers

Problem 1. $[\frac{5}{3}, \frac{7}{3})$

Problem 2. $\frac{7}{12}$

Problem 3. $\frac{11}{2}$

Problem 4.

a) $R = 1$.

b) $\frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2} x^{2n}$

c) $\arcsin x = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2 (2n+1)} x^{2n+1}$

d) The sum equals $2 \arcsin(\frac{1}{2})$, which simplifies to $\frac{\pi}{3}$

Problem 5.

a) $\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2^{2n}}{(2n)!} x^{6n} = 1 - 2x^6 + \frac{16}{24}x^{12} - \dots$

b) $\frac{2}{3} \cdot 12!$

Problem 6.

a) $e^{-3/4}$

b) $\cos \frac{\sqrt{3}}{2} - \frac{5}{8}$

Problem 7.

a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}$

b) $C + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)n!} x^{2n+1}$

c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1) \cdot 2^{2n+1} \cdot n!}$

d) 5 (for n from 0 to 4)

Problem 8.

$$3 + \frac{1}{3}(x - 4) - \frac{1}{54}(x - 4)^2 + \frac{1}{486}(x - 4)^3$$

Problem 9.

a) $3 + \frac{1}{6}(x - 9)$

b) $\frac{10}{3}$

c) $|\sqrt{11} - \frac{10}{3}| = |R_1| \leq \frac{1}{54}$

Math 227 Sample Final Examination 2 Answers

Problem 1. $[\frac{1}{3}, \frac{2}{3}]$

Problem 2. $\frac{1}{16}$

Problem 3. Center is 16, radius is 5.

Problem 4.

a) $\sum_{n=0}^{\infty} (-1)x^n$

b) $\sum_{n=0}^{\infty} \frac{-1}{2^{n+1}}x^n$

c) $\sum_{n=0}^{\infty} (1 - \frac{-1}{2^{n+1}})x^n$

Problem 5. $T_4(x) = 2x - \frac{23}{3}x^3$

Problem 6.

a) $T_2(x) = 8 + 3(x - 4) + \frac{3}{16}(x - 4)^2$

b) 9.23

c) $|4.4^{3/2} - 9.23| = |R_2| \leq \frac{1}{2000}$ (or 0.0005)

Problem 7.

a) $\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 3^{2n}}{(2n)!} x^{4n+4}$

b) $C + \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 3^{2n}}{(4n+5)(2n)!} x^{4n+5}$

c) $\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 3^{2n} \cdot 2^{4n+5}}{(4n+5)(2n)!}$

Problem 8. $T_3(x) = \frac{\pi}{4} + \frac{1}{2}(x - 1) - \frac{1}{4}(x - 1)^2$

Problem 9.

a) $\frac{\pi}{4}$

b) $e^3 - 4$

Math 227 Sample Final Examination 3 Answers

Problem 1. $(-2, 1)$

Problem 2. 1

Problem 3. Center is $-\frac{7}{2}$. Radius is 5.

Problem 4.

a) $\sum_{n=0}^{\infty} (-1)^n x^n$

b) $\frac{1}{(x+1)^2} = \sum_{n=1}^{\infty} (-1)^{n-1} n x^{n-1}$ (or $\sum_{n=0}^{\infty} (-1)^n (n+1) x^n$)

c) $\frac{1}{(x+1)^3} = \sum_{n=2}^{\infty} \frac{1}{2} (-1)^n n(n-1) x^{n-2}$

d) $\frac{7}{2}$

Problem 5. $e + 2e(x-1) + 3e(x-1)^2 + \frac{10e}{3}(x-1)^3$

Problem 6.

a) $e^{0.1} = \sum_{n=0}^{\infty} \frac{1}{10^n \cdot n!}$

b) $T_2(0.1) = 1.105$ It is the 3rd partial sum of the series from part a).

c) $n = 4$

Problem 7. (15 points)

a) $x \arctan(x^3) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{6n+4}$

b) $\int x \arctan(x^3) dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(6n+5)} x^{6n+5}$

c) $\int_0^{1/2} x \arctan(x^3) dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(6n+5)2^{6n+5}}$

Problem 8.

a) 0

b) $1 - \ln 2$