

Version 2 (11:20)

1. (15 points) Find the limit. If the limit does not exist, because it is infinite, indicate positive or negative infinity.

a)

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sin(5x)}{x^2 - 8x} \\ &= \lim_{x \rightarrow 0} \frac{\sin(5x)}{x(x-8)} \\ &= 5 \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} \cdot \frac{1}{x-8} \\ &= 5(1) \left(-\frac{1}{8} \right) \\ &= -\frac{5}{8} \end{aligned}$$

b)

$$\begin{aligned} & \lim_{x \rightarrow 2^-} \frac{x^2 + x - 6}{|x - 2|} \\ &= \lim_{x \rightarrow 2^-} \frac{(x+3)(x-2)}{|x-2|} \\ &= \lim_{x \rightarrow 2^-} \frac{(x+3)(x-2)}{-(x-2)} \\ &= \lim_{x \rightarrow 2^-} -(x+3) \\ &= -5 \end{aligned}$$

2. (15 points) Find the limit. If the limit does not exist because it is infinite, indicate positive or negative infinity.

a)

$$\begin{aligned} & \lim_{x \rightarrow -3^-} \frac{3 - \sqrt{6-x}}{x^2 + 3x} \cdot \frac{3 + \sqrt{6-x}}{3 + \sqrt{6-x}} \\ &= \lim_{x \rightarrow -3^-} \frac{9 - (6-x)}{x(x+3)(3 + \sqrt{6-x})} \\ &= \lim_{x \rightarrow -3^-} \frac{\cancel{3+x}}{x(x+3)(3 + \sqrt{6-x})} \\ &= \lim_{x \rightarrow -3^-} \frac{1}{x(3 + \sqrt{6-x})} = \boxed{-\frac{1}{18}} \end{aligned}$$

b)

$$\begin{aligned} & \lim_{x \rightarrow -3^-} \frac{3x+1}{x^2+3x} \\ &= \lim_{x \rightarrow -3^-} \frac{3x+1}{x(x+3)} \\ &= \infty \end{aligned}$$

3. (10 points) An object moves along a line so that its position at time t (seconds) is given by $s(t) = 5t^2 - 3t$ (meters).

(a) Find the velocity and acceleration of the object at time t .

$$v(t) = s'(t) = 10t - 3 \text{ m/s}$$

$$a(t) = s''(t) = 10 \text{ m/s}^2$$

(b) Find the position of the object when the velocity is 27 m/s.

$$v(t) = 10t - 3 = 27$$

$$10t = 30$$

$$t = 3 \text{ Secs.}$$

$$s(3) = 5(3)^2 - 3(3) = 36 \text{ meters}$$

4. (10 points) Find the equation of the tangent line to the graph of h at the point $(1, h(1))$.

$$h(x) = x^2 \cdot \sqrt{3x+1}$$

You may leave your answer in point-slope form.

$$h'(x) = 2x\sqrt{3x+1} + x^2 \frac{d}{dx}(3x+1)^{1/2}$$

$$= 2x\sqrt{3x+1} + \frac{x^2}{2}(3x+1)^{-1/2}(3)$$

$$= 2x\sqrt{3x+1} + \frac{3x^2}{2\sqrt{3x+1}}$$

$$m = h'(1) = 2(2) + \frac{3}{2(2)}$$

$$= \frac{19}{4}$$

$$y_T = mx + b$$

$$y_T = \frac{19}{4}x + b$$

Plug in $(1, 2)$:

$$2 = \frac{19}{4}(1) + b$$

$$b = \frac{-15}{4}$$

$$y_T = \frac{19}{4}x - \frac{15}{4}$$

5. (20 points) Function $f(x)$ is defined below.

$$f(x) = \begin{cases} x^2 + 1, & \text{if } x \leq 0 \\ \frac{\sin x}{x}, & \text{if } 0 < x < 2 \\ 0, & \text{if } 2 \leq x \end{cases}$$

a) Find, with explanation, all real numbers a , such that $f(x)$ is not continuous at a .

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} x^2 + 1 = 1$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{\sin x}{x} = \frac{\sin 2}{2}$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} 0 = 0$$

Not Cont. @ $x=2$

Since $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$

b) For each such a , find $\lim_{x \rightarrow a} f(x)$ or explain why it does not exist.

$$\lim_{x \rightarrow 0} f(x) = 1$$

$\lim_{x \rightarrow 2} f(x)$ DNE due to

$$\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$$

6. (15 points) Find the derivatives (by any method). You do not need to simplify your answer.

$$\text{a) } f(x) = x^5 \cdot \sqrt{x^4 + 3 \cos(x)}. \quad f'(x) = 5x^4 \sqrt{x^4 + 3 \cos x} + x^5 \frac{d}{dx} (x^4 + 3 \cos x)^{1/2}$$

$$f'(x) = 5x^4 \sqrt{x^4 + 3 \cos x} + x^5 \left(\frac{1}{2}\right) (x^4 + 3 \cos x)^{-1/2} (4x^3 - 3 \sin x)$$

$$\text{b) } f(x) = \frac{3x + 5}{\sin^2(x)}. \quad f'(x) = \frac{3(\sin^2 x) - (3x + 5) \frac{d}{dx} (\sin^2 x)}{\sin^4 x}$$

$$= \frac{3 \sin^2 x - (3x + 5)(2) \sin x \cos x}{\sin^4 x}$$

$$\text{c) } f(x) = \tan\left(\frac{2x-1}{x^3+x}\right). \quad f'(x) = \sec^2\left(\frac{2x-1}{x^3+x}\right) \cdot \frac{d}{dx} \left(\frac{2x-1}{x^3+x}\right)$$

$$f'(x) = \sec^2\left(\frac{2x-1}{x^3+x}\right) \cdot \left[\frac{2(x^3+x) - (2x-1)(3x^2+1)}{(x^3+x)^2} \right]$$

7. (15 points) Find the derivative of the function

$$f(x) = \frac{1}{3x+2}$$

using the limit definition of the derivative.

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{3(x+h)+2} - \frac{1}{3x+2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{3(x+h)+2} - \frac{1}{3x+2}}{h} \cdot \frac{[3(x+h)+2][3x+2]}{[3(x+h)+2][3x+2]}$$

$$= \lim_{h \rightarrow 0} \frac{3x+2 - [3(x+h)+2]}{h [3(x+h)+2][3x+2]}$$

$$= \lim_{h \rightarrow 0} \frac{-3h}{h [3(x+h)+2][3x+2]}$$

$$= \lim_{h \rightarrow 0} \frac{-3}{[3(x+h)+2][3x+2]}$$

$$= \frac{-3}{(3x+2)^2}$$