Math 224 Final Examination Fall 2015, white version

Name (print)_____

Name (sign)_____

Bing ID number _____
(Your instructor may check your ID during or after the test)

No books, no notes, no electronic devices (calculators, cell phones, smart watches, etc.) Write all your work on the test – nothing else will be graded. **You must show all your work.** Your work must be legible, and the final answers must be reasonably simplified.

On some problems you are asked to use a specific method to solve the problem (for instance, "Use the definition of the derivative to find..."). On all other problems, you may use any method we have covered. **You may not use methods that we have not covered. For instance, use of L'Hospital's Rule we receive no credit.**

Wandering Eyes Policy

You must keep your eyes on your own work at all times. If you are found looking around, you will be warned once, and only once. A second infraction may result in automatic zero on this test, and possibly a referral to the Harpur College Academic Honesty Committee.

Duration of the Test

This is a timed test designed for one class period. You will start the test when your proctor tells you to start, and you will finish the test when your proctor tells you to stop, when the class period is over.

1. (10 points) Find all critical numbers of the function

$$f(x) = \frac{x^2}{3 - x}$$

2. (10 points) Find the limits.

a) $\displaystyle \lim_{x \to -\infty} \frac{5x^3 - 3x + 1}{2x^3 + x^2}$

b) $\displaystyle \lim_{x \to -\infty} \frac{\sqrt{3x^2 - 1}}{2x - 3}$

3. (10 points) Given the derivative $f'(x)$ of the function $f(x)$, list all intervals on which $f(x)$ is increasing.

$$f'(x) = \frac{(2x-1)^2(x-4)}{x^3(x^2+3)}$$

4. (15 points) Find the absolute maximum and the absolute minimum values on the closed interval $[0, \frac{3\pi}{2}]$ of the function

$$f(x) = \sin x + \cos^2 x$$

5. (15 points) An observer is positioned 3 km away from a rocket launch pad. How fast is the distance between the rocket and the observer increasing, when the rocket is 4 km above the ground and is moving straight up at the speed of 300 m/sec?

6. (15 points) Water is leaking from a conical cup at the constant rate of 2 cm^3/min. The height of the cup is 12 cm and the radius of the top is 4 cm. How fast is the level of the water in the cup decreasing when the water is 3 cm deep? (The volume of a right circular cone is given by the formula $V = \frac{1}{3}\pi r^2 h$.)

7. (20 points) The function is given by the formula

$$f(x) = \frac{2x}{(x-2)^2}$$

Its derivative is given by

$$f'(x) = \frac{-2(x+2)}{(x-2)^3}$$

Find all of the following features of $f(x)$ and its graph (continued on the next page).

a) all intercepts with the coordinate axes;

b) all vertical and horizontal asymptotes;

c) all intervals on which the function is positive;

d) all intervals on which the function is increasing or decreasing;

e) all local maximum and minimum values;

f) all intervals on which the function is concave up or down;

g) all inflection points.

8. (5 points) Use the information you obtained in problem 7 to sketch the graph of the function from problem 7. **Your graph must match your work**. If some of the information from problem 7 appears self-contradictory, and you have no time to look for mistakes there, explain precisely what prevents you from drawing the graph.