1. [15]	2. [15]	3. [10]	4. [10]	5. [10]	6. [10]	7. [15]	8. [15]	Total: [100]

Math 224 **Exam 1** **Sept 19, 2016**

(V.1 White)

Name: Solutions *Section:*

Closed book and closed notes. *Answers must include supporting work.*

Calculators and cell phones out of sight.

1. (15 pts) Solve the following inequalities. Display your answer in interval notation.

a) $|2+3x| \leq 3$

$-3 \leq 2+3x \leq 3$

$-5 \leq 3x \leq 1$

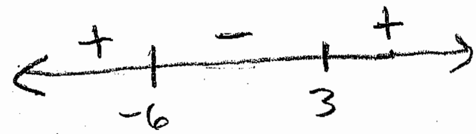
$-5/3 \leq x \leq 1/3$

$\left[-\frac{5}{3}, \frac{1}{3}\right]$

b) $x^2+3x-10 \geq 8$

$x^2+3x-18 \geq 0$

$(x+6)(x-3) \geq 0$



$x \leq -6$ or $x \geq 3$

$(-\infty, -6] \cup [3, \infty)$

2. (15 pts) Find the limit if it exists. If the limit doesn't exist explain why. **Using L'Hopital's Rule will not receive credit.**

a) $\lim\limits_{x\to 2}\dfrac{x^2-4}{x^2+x-6}$

$= \lim\limits_{x\to 2}\dfrac{(x-2)(x+2)}{(x-2)(x+3)}$

$= \lim\limits_{x\to 2}\dfrac{x+2}{x+3}$

$= \boxed{4/5}$

b) $\lim\limits_{x\to 1}\dfrac{2-x}{(x-1)^2}$

$= \boxed{\infty}$

c) $\lim\limits_{x\to 0}\dfrac{\sin(5x)}{\sin(8x)}$

$= \lim\limits_{x\to 0}\dfrac{\frac{5x\sin(5x)}{5x}}{\frac{8x\sin(8x)}{8x}}$

$= \dfrac{5}{8}\lim\limits_{x\to 0}\dfrac{\sin(5x)/5x}{\sin(8x)/8x}$

$= \dfrac{5}{8}\dfrac{\lim\limits_{x\to 0}\sin(5x)/5x}{\lim\limits_{x\to 0}\sin(8x)/8x}$

$= \dfrac{5}{8}\left(\dfrac{1}{1}\right) = \boxed{5/8}$

3. (10 pts) Use the Intermediate Value Theorem to show there is a root for the equation $x^5 - x^3 + 3x - 5 = 0$ between 1 and 2.

$P(x) = x^5 - x^3 + 3x - 5$ is continuous for all reals.

$P(1) = 1 - 1 + 3 - 5 = -2 < 0$

$P(2) = 2^5 - 2^3 + 3(2) - 5 = 25 > 0$

Hence, by Intermediate Value Thm there exists a c between 1 and 2 such that $P(c) = 0$.

4. (10 pts) Find the equation of the tangent line to the curve $y = x^2 - 2x + 3$ at $x = -1$.

$y' = 2x - 2$

$y'(-1) = 2(-1) - 2 = -4$

is the slope of the tangent line to the curve at $x = -1$.

$y(-1) = (-1)^2 - 2(-1) + 3$
$= 6$

So, $(-1, 6)$ is a pt. on the curve.

Tangent Line: $y_T = mx + b$
where $m = y'(-1) = -4$.

$y_T = -4x + b$

Plug in pt. $(-1, 6)$:

$6 = -4(-1) + b \Rightarrow b = 2$

$$\boxed{y_T = -4x + 2}$$

5. (10 pts) Find the derivative of $f(x) = \dfrac{1}{x+2}$ using the definition of derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} \cdot \frac{(x+h+2)(x+2)}{(x+h+2)(x+2)}$$

$$= \lim_{h \to 0} \frac{(x+2) - (x+h+2)}{h(x+h+2)(x+2)}$$

$$= \lim_{h \to 0} \frac{-h}{h(x+h+2)(x+2)}$$

$$= \lim_{h \to 0} \frac{-1}{(x+h+2)(x+2)} = \frac{-1}{(x+2)^2}$$

6. (10 pts) Find each x-value at which f is discontinuous. Explain your answer fully using left and right-hand limits.

$$f(x)=\begin{cases} x+2 & \text{if } x<0 \\ 2x^2 & \text{if } 0\le x\le 1 \\ 3-x & \text{if } x>1 \end{cases}$$

$\lim_{x\to 0^-} f(x) = 0+2 = 2$

$\lim_{x\to 0^+} f(x) = 2(0)^2 = 0$

$\lim_{x\to 1^-} f(x) = 2(1)^2 = 2$

$\lim_{x\to 1^+} f(x) = 3-1 = 2$

$f(x)$ is discontinuous at $x=0$ Since $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$.

7. (15 pts) Find $\frac{dy}{dx}$ for the following:

a) $y = ax^2 + \frac{b}{\sqrt{x^3}} + c$

$y = ax^2 + bx^{-3/2} + c$

$\frac{dy}{dx} = 2ax - \frac{3b}{2}x^{-5/2}$

b) $y = \sin\sqrt{x^2+1}$

$y = \sin(x^2+1)^{1/2}$

$\frac{dy}{dx} = \cos\sqrt{x^2+1}\cdot\frac{d}{dx}(x^2+1)^{1/2}$

$= \cos\sqrt{x^2+1}\cdot\frac{1}{2}(x^2+1)^{-1/2}\cdot\frac{d}{dx}(x^2+1)$

$= \frac{2x\cos\sqrt{x^2+1}}{2\sqrt{x^2+1}}$

$= \frac{x\cos\sqrt{x^2+1}}{\sqrt{x^2+1}}$

c) $y = \tan(5x)\cos x$

$\frac{dy}{dx} = \tan(5x)\frac{d}{dx}\cos x + \cos x\frac{d}{dx}\tan(5x)$

$\frac{dy}{dx} = -\sin x\tan(5x) + 5\sec^2(5x)\cos x$

8. (15 pts) Let $f(x)=\dfrac{x^2-1}{|x-1|}$. $(x\neq 1)$

Write the above function $f(x)$ as a piecewise function and find the following if they exist:

a) $\lim\limits_{x\to 1^-} f(x)$, $\lim\limits_{x\to 1^+} f(x)$, and $\lim\limits_{x\to 1} f(x)$.

b) Sketch $f(x)$.

c) Where is $f(x)$ continuous? Where is it differentiable?

d) Sketch $f'(x)$.

$$\boxed{f(x)=\begin{cases}\dfrac{(x-1)(x+1)}{x-1} & \text{if } x-1>0\\[2mm] \dfrac{(x-1)(x+1)}{-(x-1)} & \text{if } x-1<0\end{cases} = \begin{cases} x+1 & \text{if } x>1\\ -(x+1) & \text{if } x<1\end{cases}}$$

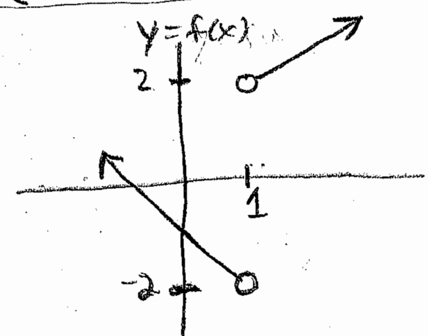
a) $\lim\limits_{x\to 1^-} f(x) = -2$

$\lim\limits_{x\to 1^+} f(x) = 2$

Since $\lim\limits_{x\to 1^-} f(x) \neq \lim\limits_{x\to 1^+} f(x)$,

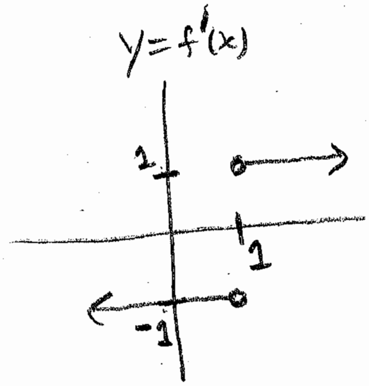
$\lim\limits_{x\to 1} f(x)$ DNE

b)



c) $f(x)$ is continuous and differentiable on $(-\infty,1)\cup(1,\infty)$

d)



$$f'(x)=\begin{cases}1 & \text{if } x>1\\ -1 & \text{if } x<1\end{cases}$$