1. [10]	2. [15]	3. [10]	4. [10]	5. [15]	6. [10]	7. [10]	8. [10]	9. [10]	Total: [100]

Math 223 **Midterm** (8:00 AM version) **Sept 28, 2015**

Name: Scott Key *Section:* ∞

Closed book and closed notes. *Answers must include supporting work.*

Calculators and cell phones out of sight.

1. (10 pts) Solve the following inequalities:

(a) $|2+3x| \le 3$ (b) $|x-1| > 3$

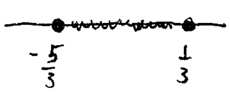
Method 1

$-3 \le 2+3x \le 3$

$-5 \le 3x \le 1$

$\frac{-5}{3} \le x \le \frac{1}{3}$

$[\frac{-5}{3}, \frac{1}{3}]$



Solution:

$[\frac{-5}{3}, \frac{-2}{3}) \cup [\frac{-2}{3}, \frac{1}{3}]$

$= [\frac{-5}{3}, \frac{1}{3}]$

Method 2 (two cases)

$2+3x \ge 0$ and $2+3x \le 3$

or

$2+3x < 0$ and $-(2+3x) \le 3$

$3x \ge -2$ and $3x \le 1$

$x \ge -\frac{2}{3}$ and $x \le \frac{1}{3}$

$[-\frac{2}{3}, \frac{1}{3}]$

$\boxed{\text{or}}$

$2+3x < 0$ and $2+3x \ge -3$

$3x < -2$ and $3x \ge -5$

$x < \frac{-2}{3}$ and $x \ge \frac{-5}{3}$

$[\frac{-5}{3}, \frac{-2}{3})$

Either

$x-1 \ge 0$ and $x-1 > 3$ ①

or

$x-1 < 0$ and $-(x-1) > 3$

① $x \ge 1$ and $x > 4$. For both, need $x > 4$ $(4, \infty)$

② $x < 1$ and $(x-1) < -3$, $x < -2$. For both, need $x < -2$ $(-\infty, -2)$

Solution: $(-\infty, -2) \cup (4, \infty)$

2. (15 pts)

(a) Determine f and g such that $h(x) = f(g(x))$ for $h(x) = \sqrt{4x - x^2}$.

(b) Find the domain of $h(x)$.

(a) $f(x) = \sqrt{x}$

$g(x) = 4x - x^2$

$h(x) = f(g(x)) = \sqrt{4x - x^2}$

(b) $\text{dom}(h) = \{x \mid 4x - x^2 \ge 0\} = ^{*} [0, 4]$

$4x - x^2 = x(4-x)$

Case 1: $x \ge 0$ and $4 - x \ge 0$

$x \ge 0$ and $4 \ge x$

$[0, 4]$

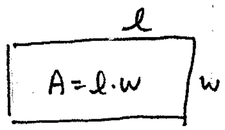
Case 2: $x < 0$ and $4 - x < 0$

$4 < x$

No real x satisfies both $x < 0$ and $4 < x$.

$*$

3. (10 pts) Given a rectangle; its length is three times its width. Express its area A as a function of its perimeter P.



Let l be the length and w the width of the rectangle; A its area and P its perimeter.

$$l = 3w$$

$$P = 2l + 2w = 2(3w) + 2w = 6w + 2w = 8w$$

$$w = \frac{P}{8}$$

$$A = l \cdot w = 3w \cdot w = 3w^2 = 3\left(\frac{P}{8}\right)^2 = \frac{3P^2}{64}$$

$$A(P) = \frac{3P^2}{64}.$$

4. (10 pts) The equation $2x^2 - 4x + 2y^2 + 8y + 1 = 0$ describes a circle. Determine the circles center and radius by completing the square.

Method 1

$$2x^2 - 4x + 2y^2 + 8y + 1 = 0$$

$$2(x^2 - 2x) + 2(y^2 + 4y) + 1 = 0$$

$$2(x^2 - 2x + \underline{?} - \underline{?}) + 2(y^2 + 4y + \underline{?} - \underline{?}) + 1 = 0$$

$$2(x^2 - 2x + 1 - 1) + 2(y^2 + 4y + 4 - 4) + 1 = 0$$

$$2[(x-1)^2 - 1] + 2[(y+2)^2 - 4] + 1 = 0$$

$$2(x-1)^2 - 2 + 2(y+2)^2 - 8 + 1 = 0$$

$$2(x-1)^2 + 2(y+2)^2 - 9 = 0$$

$$2(x-1)^2 + 2(y+2)^2 = 9$$

$$(x-1)^2 + (y+2)^2 = \frac{9}{2} = \left(\frac{3}{\sqrt{2}}\right)^2$$

Center: $(+1, -2)$

Radius: $\sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}$

Method 2

$$2x^2 - 4x + 2y^2 + 8y = -1$$

$$x^2 - 2x + y^2 + 4y = -\frac{1}{2}$$

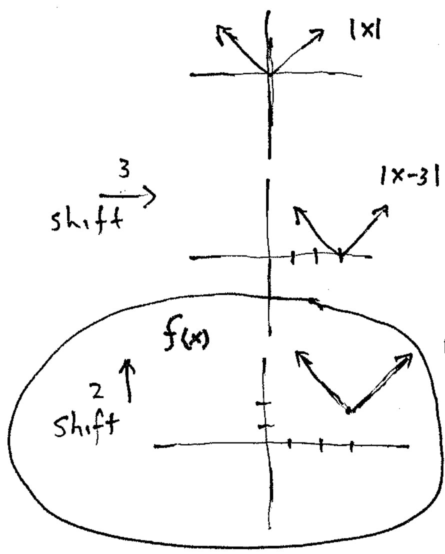
$$x^2 - 2x + \underline{1} + y^2 + 4y + \underline{4} = -\frac{1}{2} + \underline{1} + \underline{4}$$

$$(x-1)^2 + (y+2)^2 = 5 - \frac{1}{2} = \frac{9}{2}$$

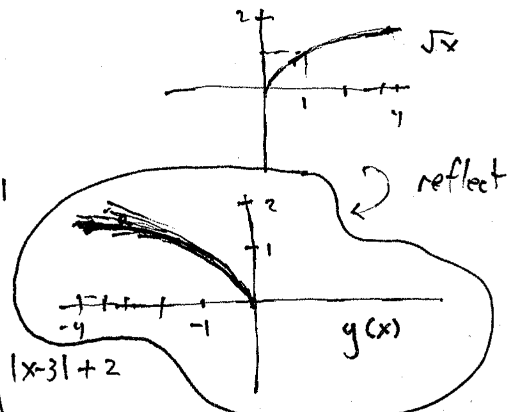
$$(x-1)^2 + (y+2)^2 = \left(\frac{3}{\sqrt{2}}\right)^2$$

5. (15 pts) Sketch the following graphs:

(a) $y = |x-3| + 2$ $= f(x)$



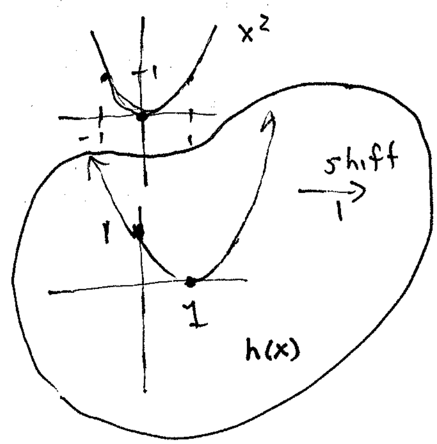
(b) $y = \sqrt{-x}$ $= g(x)$



(c) $y = x^2 - 2x + 1$ $= h(x)$

$= (x-1)(x-1)$

$= (x-1)^2$



6. (10 pts)

(a) Find a polynomial of degree 3 with zeros -3, 0, and 3.

(b) Is $x-1$ a factor of $P(x) = x^7 + 4x^6 - 2x^5 + x^4 - x^2 + 2x - 5$? Explain your answer. [Hint: use the Factor Theorem]

(a) Since -3, 0, and 3 are roots, $(x+3)$, $(x-0)$, and $(x-3)$ must be factors. Take $p(x) = (x+3) \cdot x \cdot (x-3)$. ← Full credit

$$\left.\begin{aligned} &= x(x+3)(x-3) \\ &= x(x^2-9) \\ &= x^3 - 9x \end{aligned}\right\} \text{ Not needed; usually more complicated!}$$

(b) Observe: $P(1) = 1^7 + 4 \cdot 1^6 - 2 \cdot 1^5 + 1^4 - 1^2 + 2 \cdot 1 - 5$

$= 1 + 4 - 2 + 1 - 1 + 2 - 5$

$= 0.$

The Factor Theorem tells us that $x-1$ is a factor of $P(x)$. Yes.

7. (10 pts) Find the quotient and the remainder using long division.

$$\frac{3x^4-5x^3-20x-5}{x^2+x+3}$$

$$
\begin{array}{r|l}
 & 3x^2 - 8x - 1 \\
x^2+x+3 & 3x^4 - 5x^3 + 0x^2 - 20x - 5 \\
 & \underline{3x^4 + 3x^3 + 9x^2} \\
 & \quad -8x^3 - 9x^2 - 20x - 5 \\
 & \quad \underline{-8x^3 - 8x^2 - 24x} \\
 & \qquad -x^2 + 4x - 5 \\
 & \qquad \underline{-x^2 - x - 3} \\
 & \qquad\quad 5x - 2
\end{array}
$$

$\leftarrow$ Quotient: $3x^2-8x-1$

$\leftarrow$ Remainder: $5x-2$

8. (10 pts) Write the following polynomial in factored form.

$$P(x)=x^3+3x^2-x-3$$

Possible integer roots: $+1, -1, +3, -3$.

$P(1) = 1+3-1-3 = 0$, so $(x-1)$ is a factor.

$P(-1) = -1+3+1-3 = 0$, so $(x+1)$ is a factor.

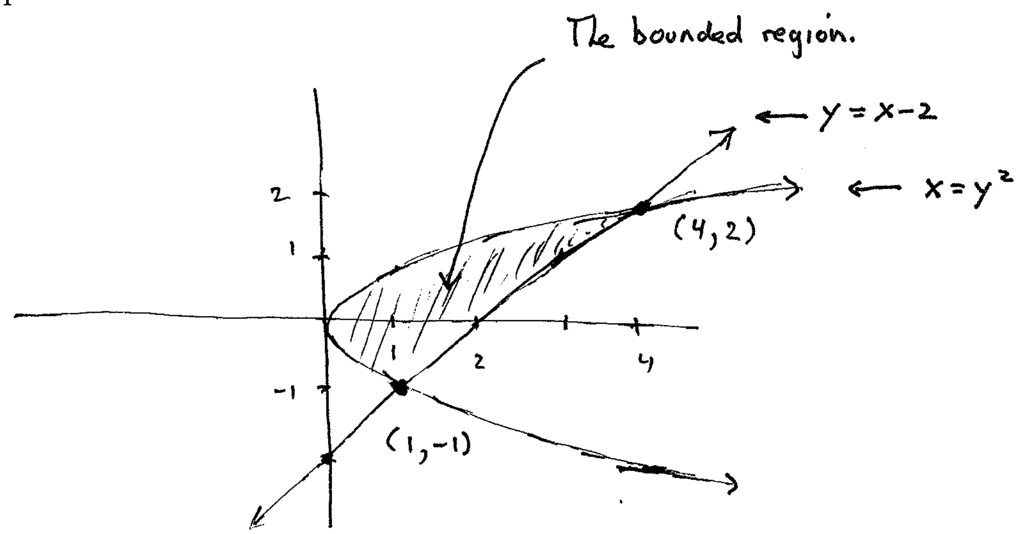
$P(-3) = (-3)^3 + 3(-3)^2 - (-3) - 3$

$= -27+27+3-3 = 0$, so $(x+3)$ is a factor.

$P(x) = (x-1)(x+1)(x+3)$.

There were other ways to solve this problem.

9. (10 pts) Sketch the region bounded by the parabola $x = y^2$ and the line $y = x - 2$ and label their points of intersection.



$$x = (x-2)^2$$
$$= x^2 - 4x + 4$$
$$0 = x^2 - 5x + 4$$
$$= (x-4)(x-1)$$

$x = 4$ or $x = 1$

$y = 4-2$ $\quad$ $y = 1-2$

$= 2$ $\quad$ $= -1$

$(4,2)$ $\quad$ $(1,-1)$ ← Intersection points, i.e., points that lie on both equations' graphs.

1. [10]	2. [15]	3. [10]	4. [10]	5. [15]	6. [10]	7. [10]	8. [10]	9. [10]	Total: [100]

Math 223 **Midterm** (11:20 AM version) **Sept 28, 2015**

Name: Frank Key *Section:* ∞

Closed book and closed notes. *Answers must include supporting work.*

Calculators and cell phones out of sight.

1. (10 pts) Solve the following inequalities:

(a) $|1-2x| \le 3$ (b) $|x-2| > 1$

Method 1

$-3 \le 1-2x \le 3$

$-4 \le -2x \le 2$

$2 \ge x \ge -1$

$[-1, 2]$

-1 2

Method 2

Case 1: $1-2x \ge 0$ and $1-2x \le 3$

$-2x \ge -1$ " $-2x \le 2$

$x \le \frac{1}{2}$ " $x \ge -1$

$[-1, \frac{1}{2}]$

or

Case 2: $1-2x < 0$ and $-(1-2x) \le 3$

$-2x < -1$ " $1-2x \ge -3$

$x > \frac{1}{2}$ " $-2x \ge -4$

$x \le 2$

$(\frac{1}{2}, 2]$

Solution: $[-1, \frac{1}{2}] \cup (\frac{1}{2}, 2]$

$= [-1, 2]$.

Case 1: $x-2 \ge 0$ and $x-2 > 1$

$x \ge 2$ and $x > 3$

$(3, \infty)$

or

Case 2: $x-2 < 0$ and $-(x-2) > 1$

$x < 2$ and $x-2 < -1$

$x < 1$

$(-\infty, 1)$

Solution $(-\infty, 1) \cup (3, \infty)$.

2. (15 pts)

(a) Determine f and g such that $h(x) = f(g(x))$ for $h(x) = \sqrt{2x-x^2}$.

(b) Find the domain of $h(x)$.

(a) * $g(x) = 2x - x^2$

$f(x) = \sqrt{x}$

$h(x) = f(g(x)) = \sqrt{2x-x^2}$

The are, of course, other solutions, for example

$g(x) = 2x$

$f(x) = \sqrt{x - (\frac{x}{2})^2}$

but the first solution * is the most obvious and useful.

(b) $\text{dom}(h) = \{x \mid 2x - x^2 \ge 0\}$

$2x - x^2 \ge 0$

$x(2-x) \ge 0$

Case 1: $x \ge 0$ and $2-x \ge 0$

$x \ge 0$ " $2 \ge x$

$0 \le x \le 2$

$[0, 2]$

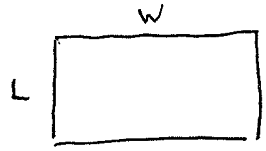
or

Case 2: $x<0$ and $2-x<0$

$x<0$ and $2<x$

No x satisfies both.

3. (10 pts) A rectangle has perimeter 12 m. Express the area A of the rectangle as a function of the length, L, of one of its sides.



$P = 2L + 2W = 12.$

$A = L \cdot W.$

Let L be the length of one side, W the length of the other; A the area and P the perimeter.

$A = L \cdot W$

$A(L) = L(6-L)$

$= 6L - L^2$ ← Expanding not required and often discouraged.

$2W = 12 - 2L$

$W = \frac{1}{2}(12-2L)$

$= 6 - L$

4. (10 pts) The equation $x^2 + y^2 - 4x + 10y + 13 = 0$ describes a circle. Determine the circles center and radius by completing the square.

Method 1

$x^2 - 4x + y^2 + 10y = -13$

$x^2 - 4x + 4 + y^2 + 10y + 25 = -13 + 4 + 25$

$(x-2)^2 + (y+5)^2 = 16 = 4^2$

Center: $(2, -5)$

Radius: 4

Method 2

$x^2 + y^2 - 4x + 10y + 13 = 0$

$x^2 - 4x + 4 - 4 + y^2 + 10y + 25 - 25 + 13 = 0$

$(x-2)^2 + (y+5)^2 - 4 - 25 + 13 = 0$

$(x-2)^2 + (y+5)^2 - 16 = 0$

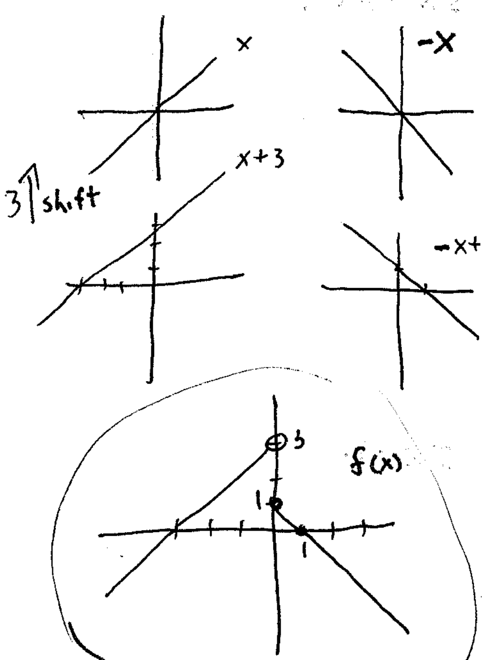
$(x-2)^2 + (y+5)^2 = 16$

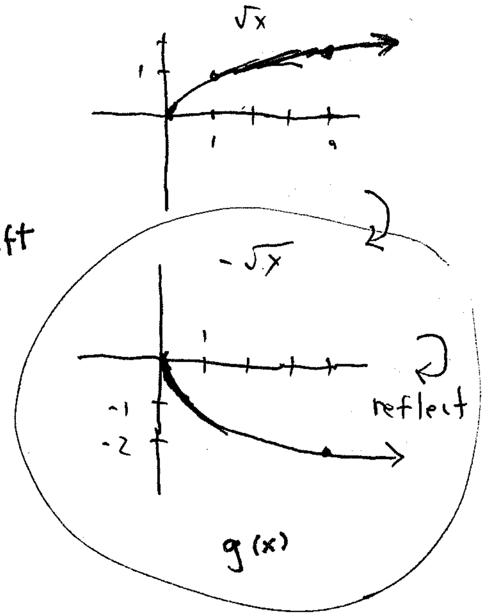
Center: $(2, -5)$

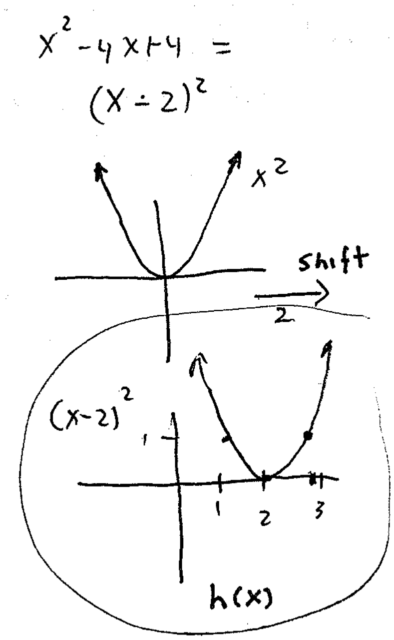
Radius: 4

5. (15 pts) Sketch the following functions:

(a) $f(x)=\begin{cases} x+3 & \text{if } x<0 \\ 1-x & \text{if } x\geq 0 \end{cases}$ (b) $y=-\sqrt{x}$ $= g(x)$ (c) $y=x^2-4x+4$ $= h(x)$







6. (10 pts)

(a) Find a polynomial of degree 3 with zeros -2, 0, and 2.

(b) Is $x-1$ a factor of $P(x)=x^7+4x^6-2x^5+x^4-x^2+2x-5$? Explain your answer. [Hint: use the Factor Theorem]

By the factor theorem, $(x-(-2))$, $(x-0)$, and $(x-2)$ must be factors.

(a) $p(x) = (x-(-2))(x-0)(x-2)$

$= (x+2)\cdot x\cdot(x-2)$ This clearly has degree 3.

$= x(x-2)(x+2)$

$= x(x^2-4)$ } Not needed; usually more complicated!

$= x^3-4x$

(b) Since $P(1) = 1^7+4\cdot 1^6-2\cdot 1^5+1^4-1^2+2\cdot 1-5$

$= 1+4-2+1-1+2-5$

$= 0$, by the Factor Theorem $(x-1)$ is a factor of $p(x)$. Yes.

7. (10 pts) Find the quotient and the remainder using long division.

$$\frac{3x^4 - 5x^3 - 20x - 5}{x^2 + x + 3}$$

$$
\begin{array}{r l}
 & 3x^2 - 8x - 1 \\
x^2+x+3 \,\big) & 3x^4 - 5x^3 + 0x^2 - 20x - 5 \\
 & 3x^4 + 3x^3 + 9x^2 \\
\hline
 & -8x^3 - 9x^2 - 20x - 5 \\
 & -8x^3 - 8x^2 - 24x \\
\hline
 & -x^2 + 4x - 5 \\
 & -x^2 - x - 3 \\
\hline
 & 5x - 2
\end{array}
$$

← Quotient: $3x^2 - 8x - 1$

← Remainder: $5x - 2$

8. (10 pts) Factor completely the following polynomial.

$$P(x) = x^3 + 3x^2 - x - 3$$

Possible integer roots: $1, -1, 3, -3$.

Observe: $P(1) = 1^3 + 3\cdot1^2 - 1 - 3 = 1 + 3 - 1 - 3 = 0$ so $(x-1)$ is a factor.

$P(-1) = (-1)^3 + 3\cdot(-1)^2 - (-1) - 3 = (-1) + 3 + 1 - 3 = 0$ so $(x+1)$ is a factor.

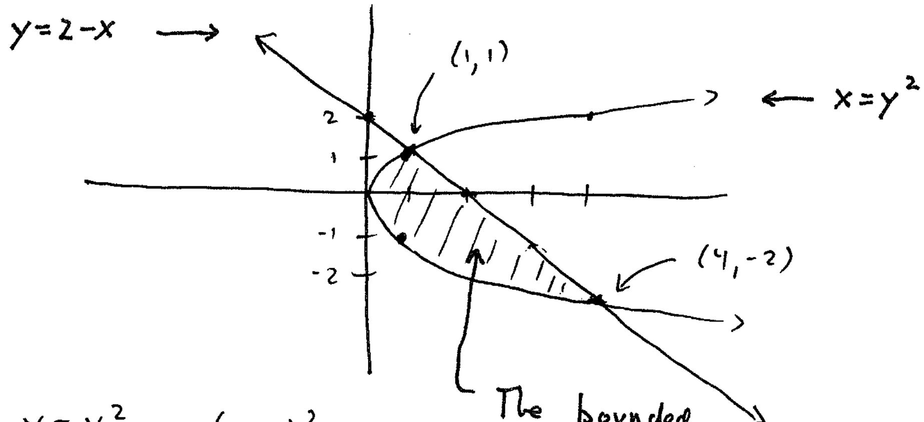
$P(-3) = (-3)^3 + 3(-3)^2 - (-3) - 3$

$= -27 + 27 + 3 - 3 = 0$ so $(x-(-3)) = (x+3)$ is a factor.

$p(x) = (x-1)(x+1)(x+3)$.

There were other ways to solve this problem.

9. (10 pts) Sketch the region bounded by the parabola $x = y^2$ and the line $y = 2 - x$ and label their points of intersection.



$$x = y^2 = (2-x)^2$$
$$= 4 - 4x + x^2$$

$$x^2 - 4x + 4 = x$$
$$x^2 - 5x + 4 = 0$$
$$\shortparallel$$
$$(x-4)(x-1)$$

$x = 4$ or $x = 1$

Curves ~~Lines~~ meet when $x = 4$ and when $x = 1$

$y = 2 - 4 = -2$ $\qquad$ $y = 2 - 1 = 1$

$$\underbrace{(4,-2) \qquad (1,1)}$$

Intersection points – points that lie on the graphs of both equations.