**Math 223**
**Introduction to Calculus**

**(White)**

**Final Exam**
**October 17, 2016**

Name (print): Frank Quay Name (sign): Frank Quay Section: $\infty$

## Instructions

Write clear, careful, neat solutions to the following questions in the space provided. If you need extra space, use the back of the previous page.

When a computation is required, show *all* work necessary to obtain the result. **You must show all your work. Your work must be legible, and the final answers must be reasonably simplified.**

No books, no notes, no electronic devices (calculators, cell phones, smart watches, etc.) **All calculators and cell phones must be put away completely out of sight.** Write all your work on the test – nothing else will be graded.

On some problems you are asked to use a specific method to solve the problem; on all other problems, you may use any method we have covered. **You may not use methods that we have not covered.**

## Wandering Eyes Policy

You must keep your eyes on your own work at all times. If you are found looking around, you will be warned once, and only once. A second infraction may result in automatic zero on this test, and possibly a referral to the Harpur College Academic Honesty Committee.

## Duration of the Test

This is a timed test designed for one class period. You will start the test when your instructor tells the entire section to start, and you will finish the test when your instructor tells you to stop, when the class period is over.

| 1. [15] | 2. [10] | 3. [15] | 4. [5] | 5. [15] | 6. [10] | 7. [10] | 8. [10] | 9. [15] | Total: [105] | |
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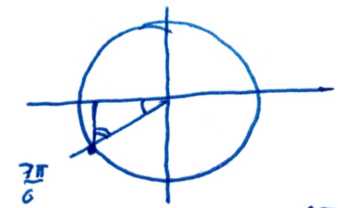
**Problem 1.** [15 pts] Compute the values of the following trigonemetric functions at the given points:

(a) $\cos(\frac{7\pi}{6}) = -\frac{\sqrt{3}}{2}$

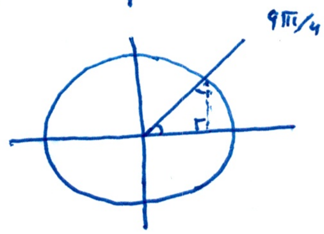


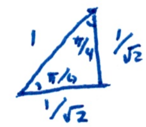


(b) $\tan(\frac{9\pi}{4}) = 1$

$\sin\left(\frac{9\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\cos\left(\frac{9\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$



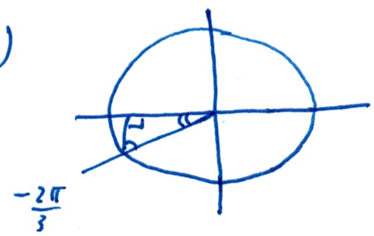



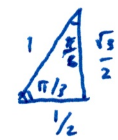

(c) $\sec(-\frac{2\pi}{3}) =$

$\cos\left(\frac{-2\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right)$

$= -\frac{1}{2}$

$= \frac{1}{\cos\left(\frac{4\pi}{3}\right)} = -2$




**Problem 2.** [10 pts] Expand as a sum or difference of logarithms and simplify as much as possible:

$$\log_2\left(\frac{x^2+2x+8}{16(x^2+4)(x+2)}\right) = \cdots$$

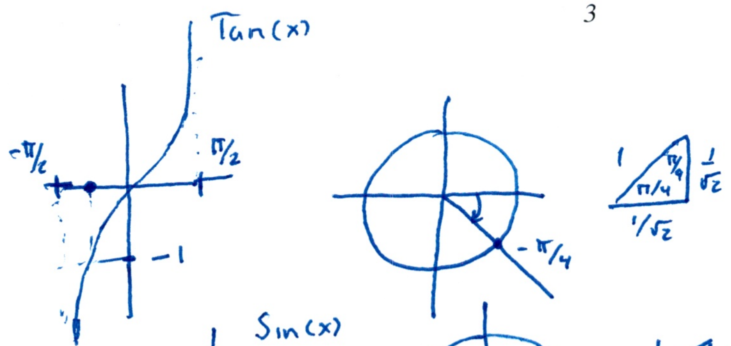
$$\cdots = \log_2(x^2+2x+8) - \left[\log_2(16) + \log_2(x^2+4) + \log_2(x+2)\right]$$

$$= \log_2(x^2+2x+8) - \log_2(16) - \log_2(x^2+4) - \log_2(x+2)$$

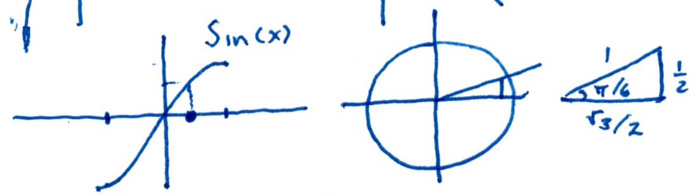
$$= \log_2(x^2+2x+8) - 4 - \log_2(x^2+4) - \log_2(x+2)$$

**Problem 3.** [15 pts] Find the exact value of each expression.

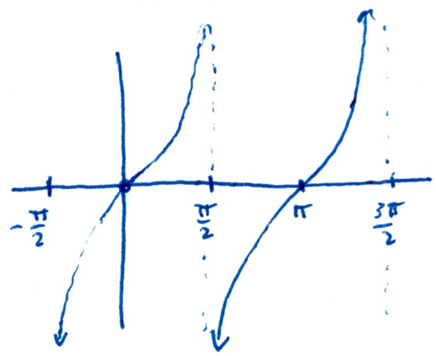
(a) $\arctan(-1)$ $= -\dfrac{\pi}{4}$




(b) $\arcsin(\frac{1}{2})$ $= \dfrac{\pi}{6}$




(c) $e^{\ln(27)}$ $= 27$




(d) $\arctan(\tan(\pi))$ $= \arctan(0)$

$= 0$

(e) $\log_{25}(\frac{1}{5})$ $= -\dfrac{1}{2}$ because $25^{-1/2} = \dfrac{1}{\sqrt{25}} = \dfrac{1}{5}$

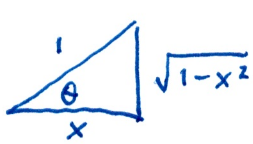
**Problem 4.** [5 pts] Rewrite the expression as an algebraic expression in $x$:

$\tan(\arccos(x)) =$ $\dfrac{\sqrt{1-x^2}}{x}$

<u>Method 1</u> $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)} = \dfrac{\sqrt{1-\cos^2(\theta)}}{\cos(\theta)}$ for $\theta \in (0, \pi)$

$$\tan(\arccos(x)) = \frac{\sqrt{1-\cos^2(\arccos(x))}}{\cos(\arccos(x))} = \frac{\sqrt{1-x^2}}{x}$$

<u>Method 2</u> $\theta = \arccos(x)$ $\tan(\arccos(x)) =$




$\tan(\theta) = \dfrac{\sqrt{1-x^2}}{x}$

**Problem 5.** [15 pts] Solve each equation below:

(a) $\ln(3x+4) = 42$

$$3x + 4 = e^{42}$$

$$3x = e^{42} - 4$$

$$x = \frac{e^{42} - 4}{3}$$

(b) $e^{x^2-1} = 8$

$$x^2 - 1 = \ln(e^{x^2-1}) = \ln(8)$$

$$x^2 = \ln(8) + 1$$

$$x = \sqrt{\ln(8)+1} \quad \text{or} \quad -\sqrt{\ln(8)+1}$$

(Note: $\ln(8) + 1 > 0$ because $8 > 1$.)

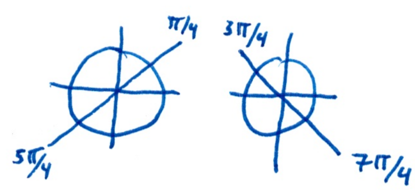
(c) $6\tan^2(x) - 2 = 0$ in the interval $[0, 2\pi]$.

$$2(\tan^2(x) - 1) = 0$$

$$2(\tan(x)-1)(\tan(x)+1) = 0$$

$$\tan(x) = 1 \quad \text{or} \quad \tan(x) = -1$$

$$x = \frac{\pi}{4} \text{ or } \frac{5\pi}{4} \qquad x = \frac{3\pi}{4} \text{ or } \frac{7\pi}{4}$$




**Problem 6.** [10 pts] A sum of \$8,000 is invested in a bank that pays interest *continuously* at an annual rate of 5%. If the sum is left in the bank to accumulate interest, how many years will it take for the account to reach \$32,000? [Your answer should be written in a form that could be entered into a calculator for an approximation. Do not attempt to simplify.]

$$P(t) = 8000e^{.05t}$$ (t measured in years).

$$32000 = 8000e^{.05t}$$

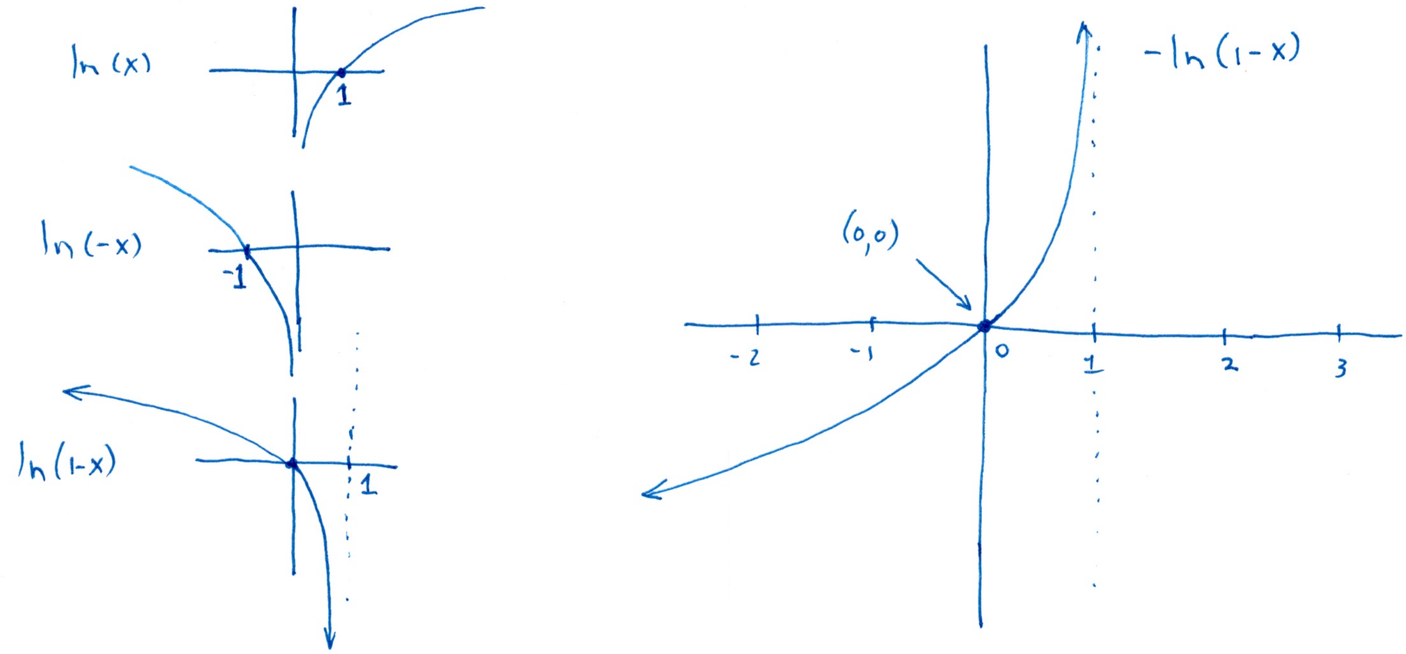
$$e^{.05t} = 4$$

$$.05t = \ln(4) \quad (\text{or } \log(4))$$

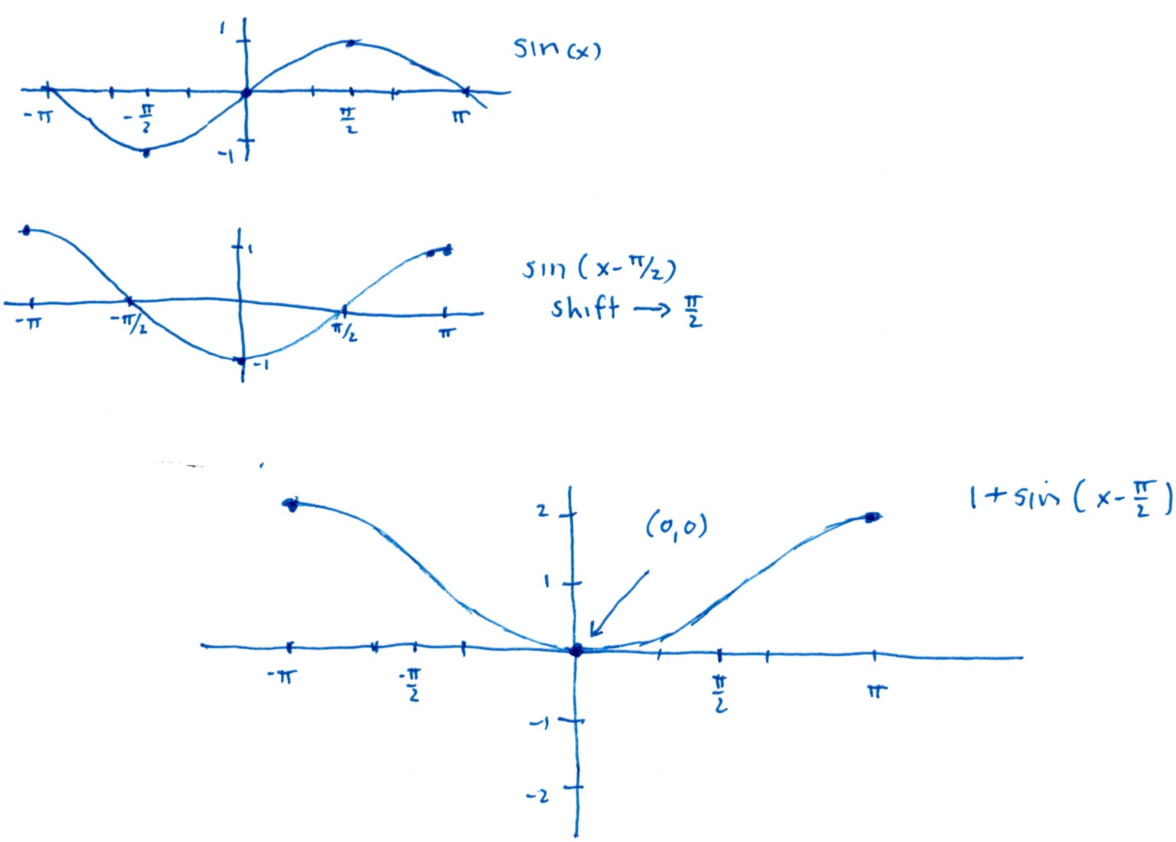
$$t = \frac{\ln(4)}{.05} = 20\ln(4) \text{ years.}$$

**Problem 7.** [10 pts] Sketch graphs of the following functions. Be sure to label the $x$- and $y$-intercepts and all asymptotes and to put a *scale* on your axes!

(a) $-\ln(1-x)$




(b) $1 + \sin(x - \frac{\pi}{2})$ over the interval $[-\pi, \pi]$



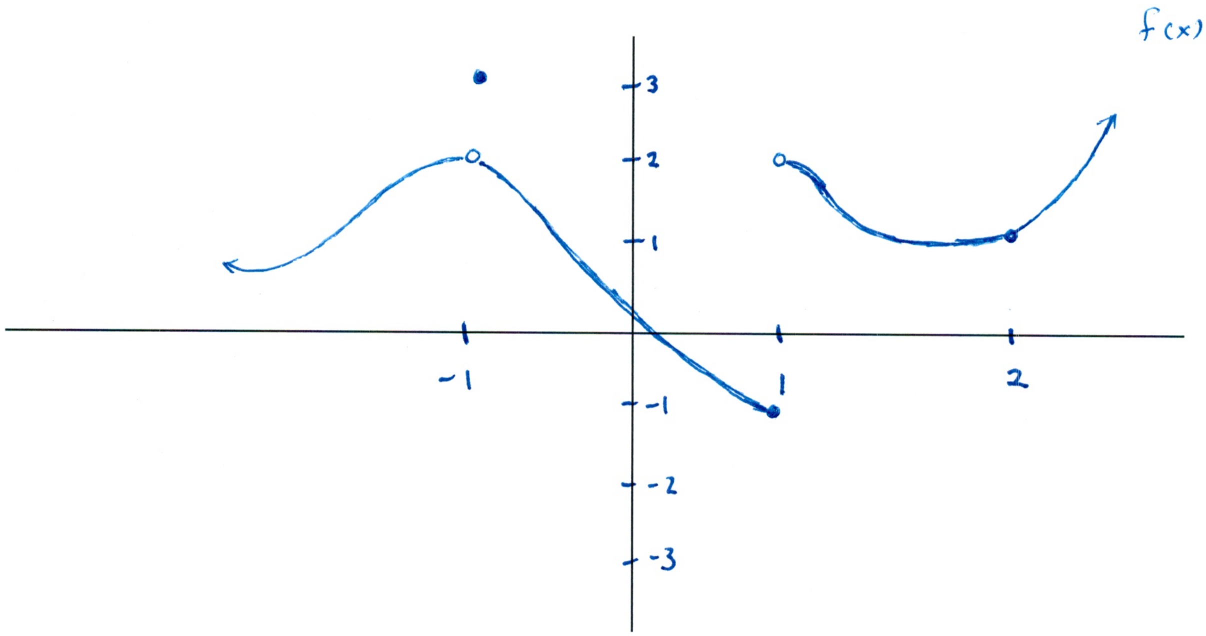
**Problem 8.** [10 pts]

(a) On the axes below, sketch a function $f(x)$ with the following properties. (If any are *impossible*, say so and explain briefly why.)

$$f(-1) = 3 \qquad \lim_{x\to -1} f(x) = 2$$

$$f(1) = -1 \qquad \lim_{x\to 1^-} f(x) = -1 \qquad \lim_{x\to 1+} f(x) = 2$$

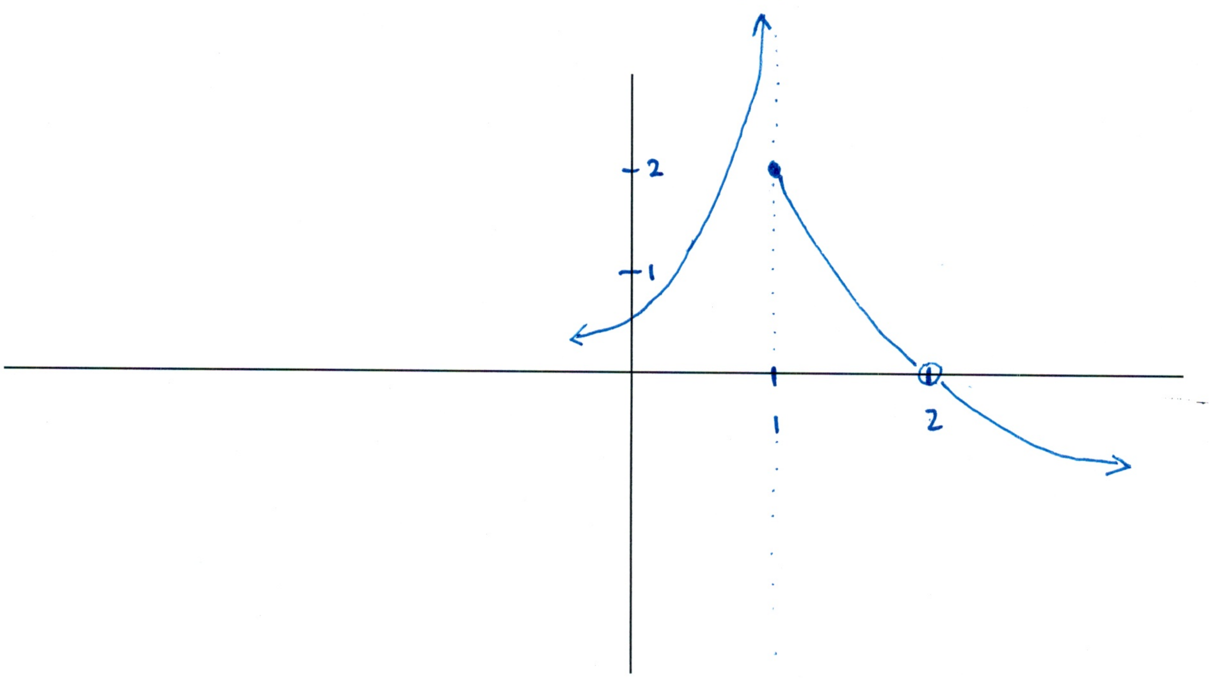
$$f(2) = 1 \qquad f(x) \text{ is continuous at } 2$$




(b) On the axes below, sketch a function $g(x)$ with the following properties. (If any are *impossible*, say so and explain briefly why.)

$$g(1) = 2 \qquad \lim_{x\to 1^-} g(x) = \infty \qquad \lim_{x\to 1+} g(x) = 2$$

$$g(2) \text{ is undefined} \qquad \lim_{x\to 2^-} g(x) = 0 \qquad \lim_{x\to 2+} g(x) = 0$$



**Problem 9.** [15 pts] Compute the following limits.

(a) $\lim_{x\to 6} \dfrac{\sqrt{x+3}-3}{x-6} =$

$$= \lim_{x\to 6}\left(\frac{\sqrt{x+3}-3}{x-6}\right)\left(\frac{\sqrt{x+3}+3}{\sqrt{x+3}+3}\right) \quad \text{because } \frac{\sqrt{x+3}+3}{\sqrt{x+3}+3} = 1$$

$$= \lim_{x\to 6} \frac{(x+3)-9}{(x-6)(\sqrt{x+3}+3)}$$

$$= \lim_{x\to 6} \frac{x-6}{(x-6)(\sqrt{x+3}+3)}$$

$$= \lim_{x\to 6} \frac{1}{\sqrt{x+3}+3} \quad \left(\text{because } \frac{x-6}{x-6} = 1 \text{ near } x=6\right)$$

(limits are local)

$$= \frac{\lim_{x\to 6} 1}{\lim_{x\to 6}\sqrt{x+3} + \lim_{x\to 6} 3} \quad \text{(quotient theorem)}$$

$$= \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

(b) $\lim_{x\to 5} \dfrac{x^2-3x-10}{x-5} =$

$$= \lim_{x\to 5} \frac{(x-5)(x+2)}{x-5}$$

$$= \lim_{x\to 5} x+2 \quad \left(\text{because } \frac{x-5}{x-5} = 1 \text{ near } 5\right)$$

(limits are local)

$$= \lim_{x\to 5} x + \lim_{x\to 5} 2 \quad \text{(sum theorem)}$$

$$= 5 + 2 = 7$$

(c) $\lim_{x\to 5^-} \dfrac{25-x^2}{|x-5|} =$

$$= \lim_{x\to 5} \frac{25-x^2}{-(x-5)} \quad \left(\text{because } |x-5| = -(x-5) \text{ when } x<5\right)$$

$$= \lim_{x\to 5} \frac{(5-x)(5+x)}{5-x}$$

$$= \lim_{x\to 5} 5+x \quad \left(\text{because } \frac{5-x}{5-x} = 1 \text{ when } x \text{ is near } 5\right)$$

(limits are local)

$$= \lim_{x\to 5} 5 + \lim_{x\to 5} x$$

$$= 5 + 5 = 10$$